Analysis of Runaway Electron Synchrotron Emission in Alcator C-Mod

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Monday, June 6, 2016

Runaway Electron Meeting 2016

Pertuis, France

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Runaway electrons in C-Mod

Alcator C-Mod plasma parameters:

B_{tor} = 2 – 8 T

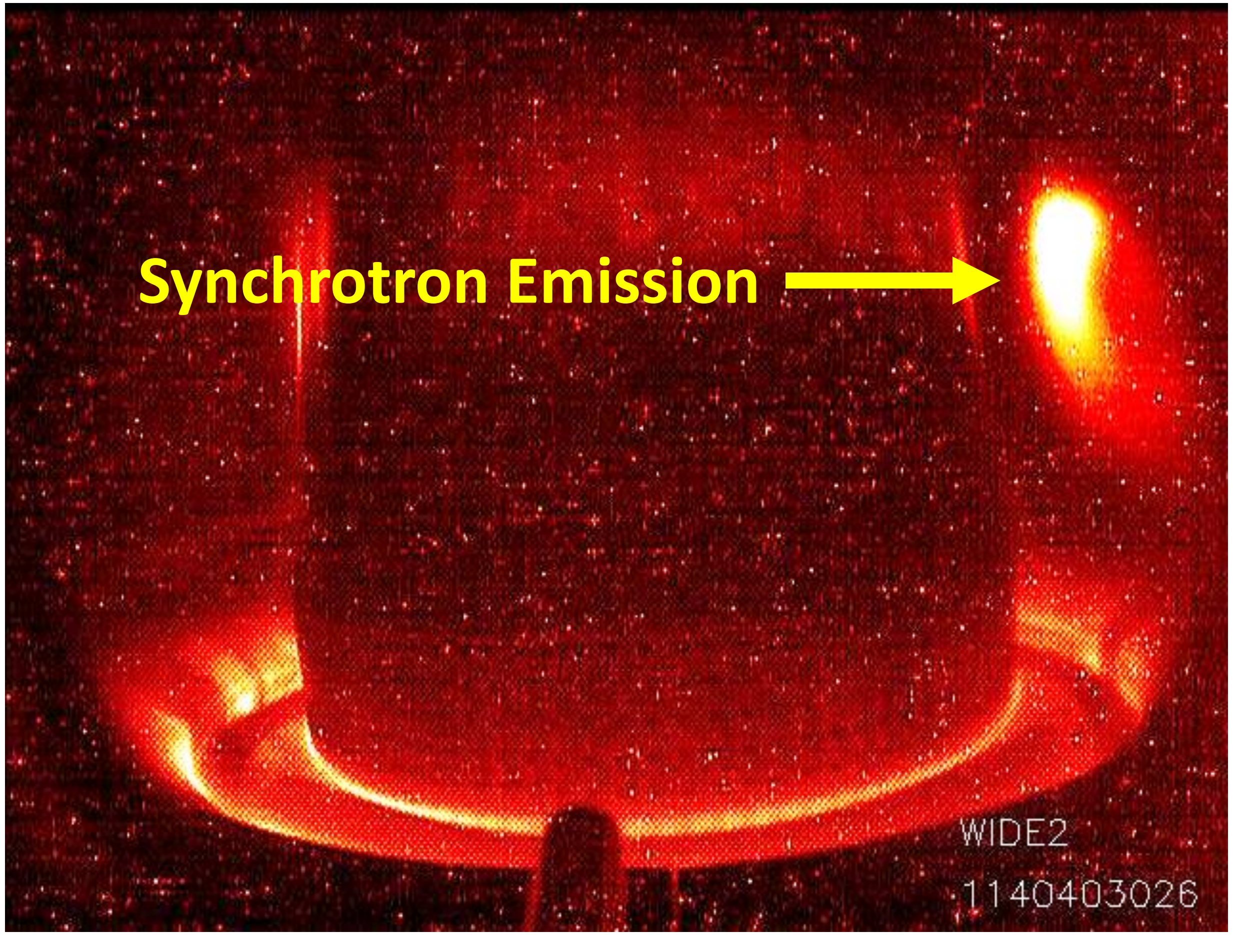
I_p = 0.5 – 2 MA

$\bar{n}_e = 0.2 – 4 \cdot 10^{20}\ m^{-3}$

T_{e0} = 1 – 8 keV

R = 0.68 m, a = 0.22 m

Synchrotron radiation (SR) can be in the visible/near-infrared range (300-1000 nm).



Camera view inside Alcator C-Mod.

Motivation

Q: From SR, can we distinguish a **mono-energetic** (and mono-pitch) RE distribution from a **continuum distribution** of energies and pitches?

- Recent studies [1-3] have predicted that REs will accelerate to a maximum energy at which the radiative force and collisional friction balances the electric force, forming a “bump” on the tail of the energy distribution function.
- Others [4,5] suggest that a broader distribution contributes to the SR spectra.
- Knowing the maximum energy of REs – as determined by the distribution – can have important implications for RE mitigation in fusion devices.

[1] P. Aleynikov, et al. Phys. Rev. Lett. 114, 155001 (2015).
[2] J. Decker, et al. Plasma Phys. Contr. Fusion 58, 025016 (2016).
[3] E. Hirvijoki, et al. J. Plasma Phys., vol. 81, 47810502 (2015).
[4] A. Stahl, et al. Phys. Plasmas 20, 093302 (2013).
[5] M. Landreman, et al. Computer Physics Communications 185, 847 (2014).

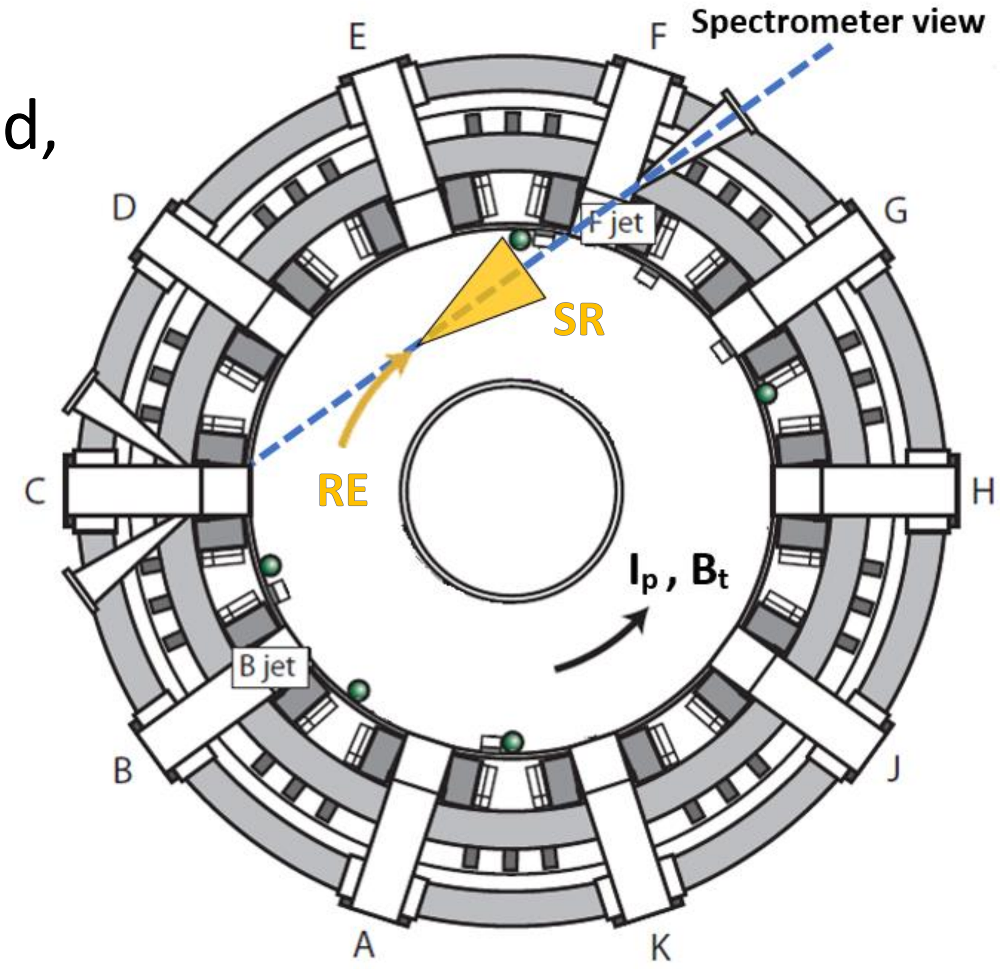
Motivation

Q: From SR, can we distinguish a **mono-energetic** (and mono-pitch) RE distribution from a **continuum distribution** of energies and pitches?

A: Not yet…

Experimental setup

Data is collected using an absolutely-calibrated spectrometer installed on C-Mod, with spectral range of ~350-1020 nm.



Toroidal cross section of C-Mod

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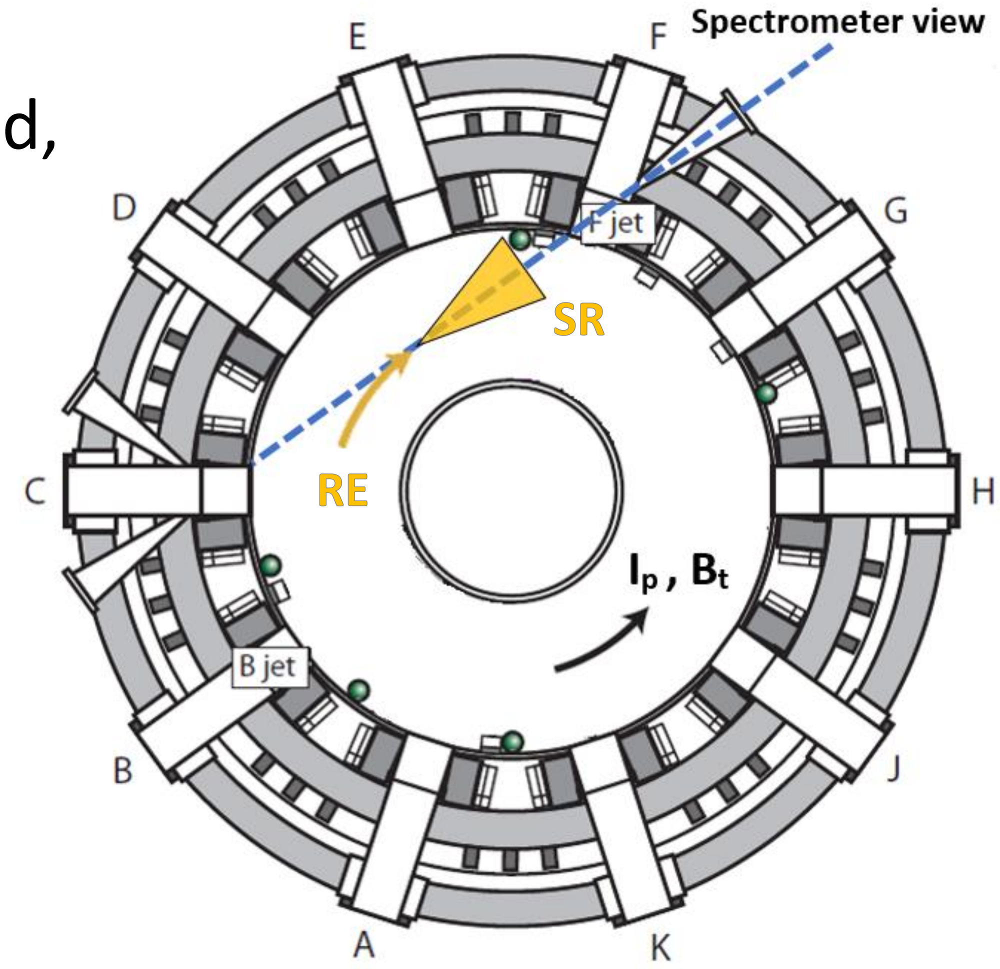
View outside vessel

View inside vessel



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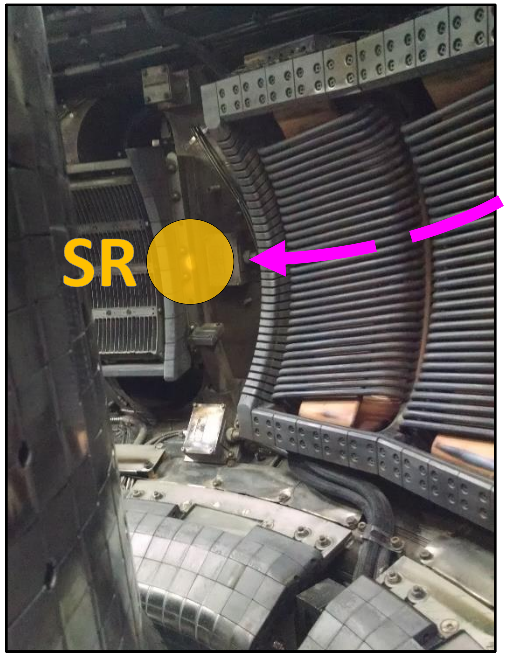

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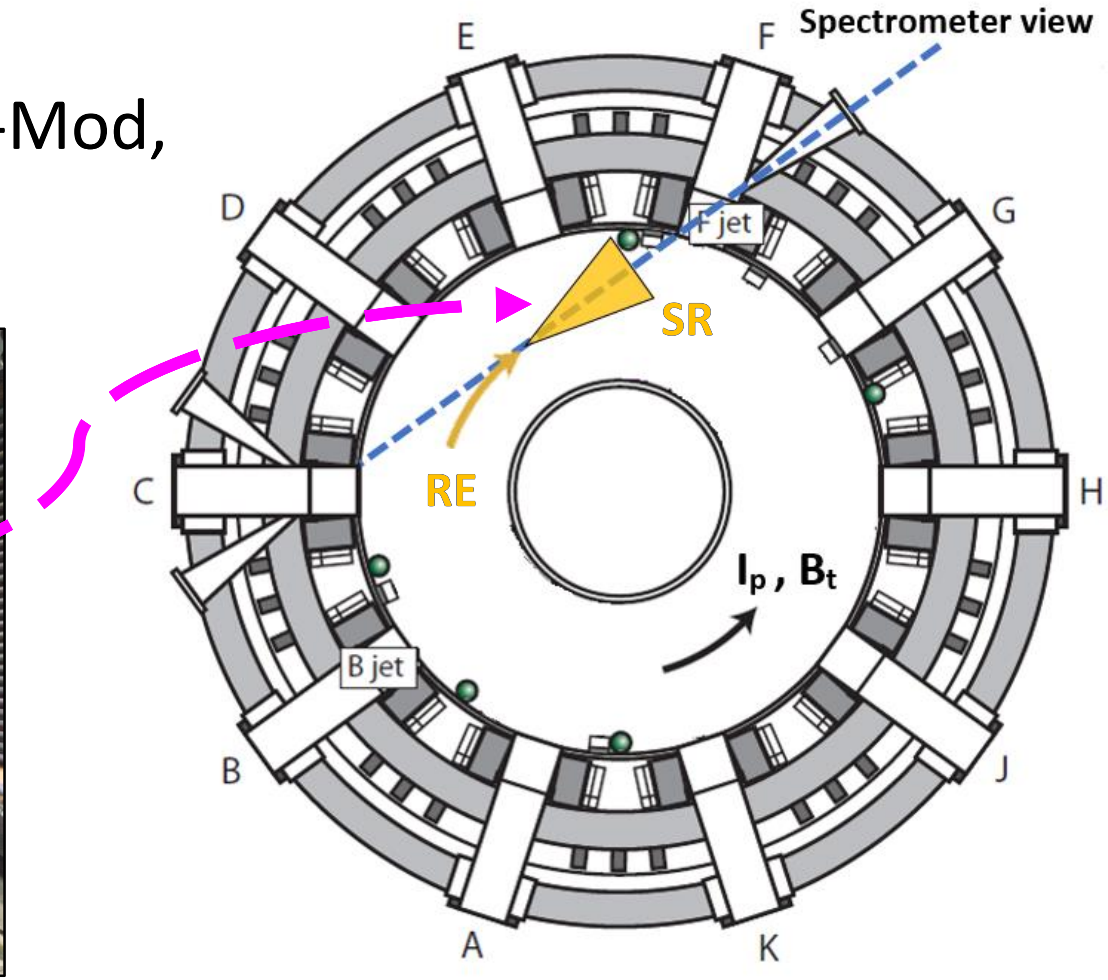

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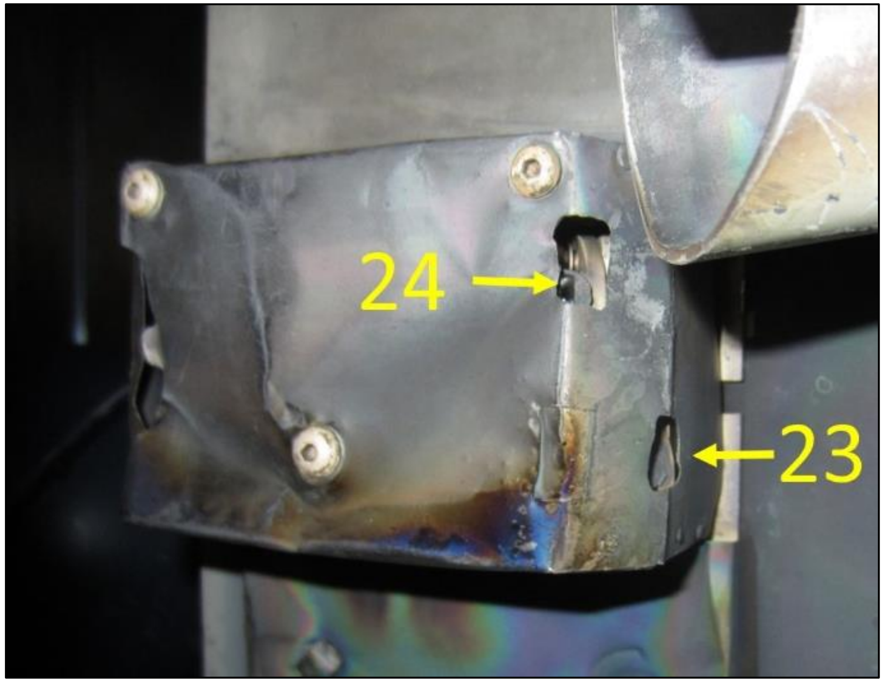
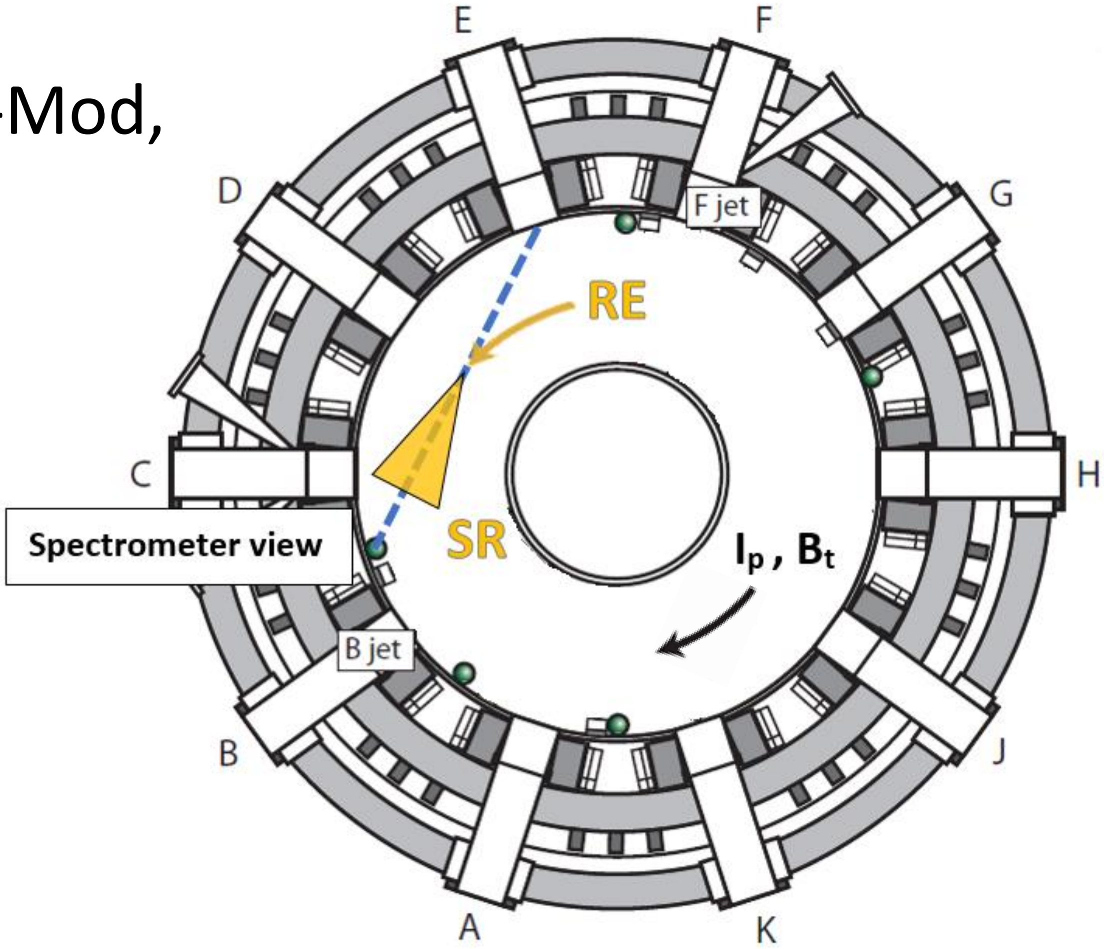
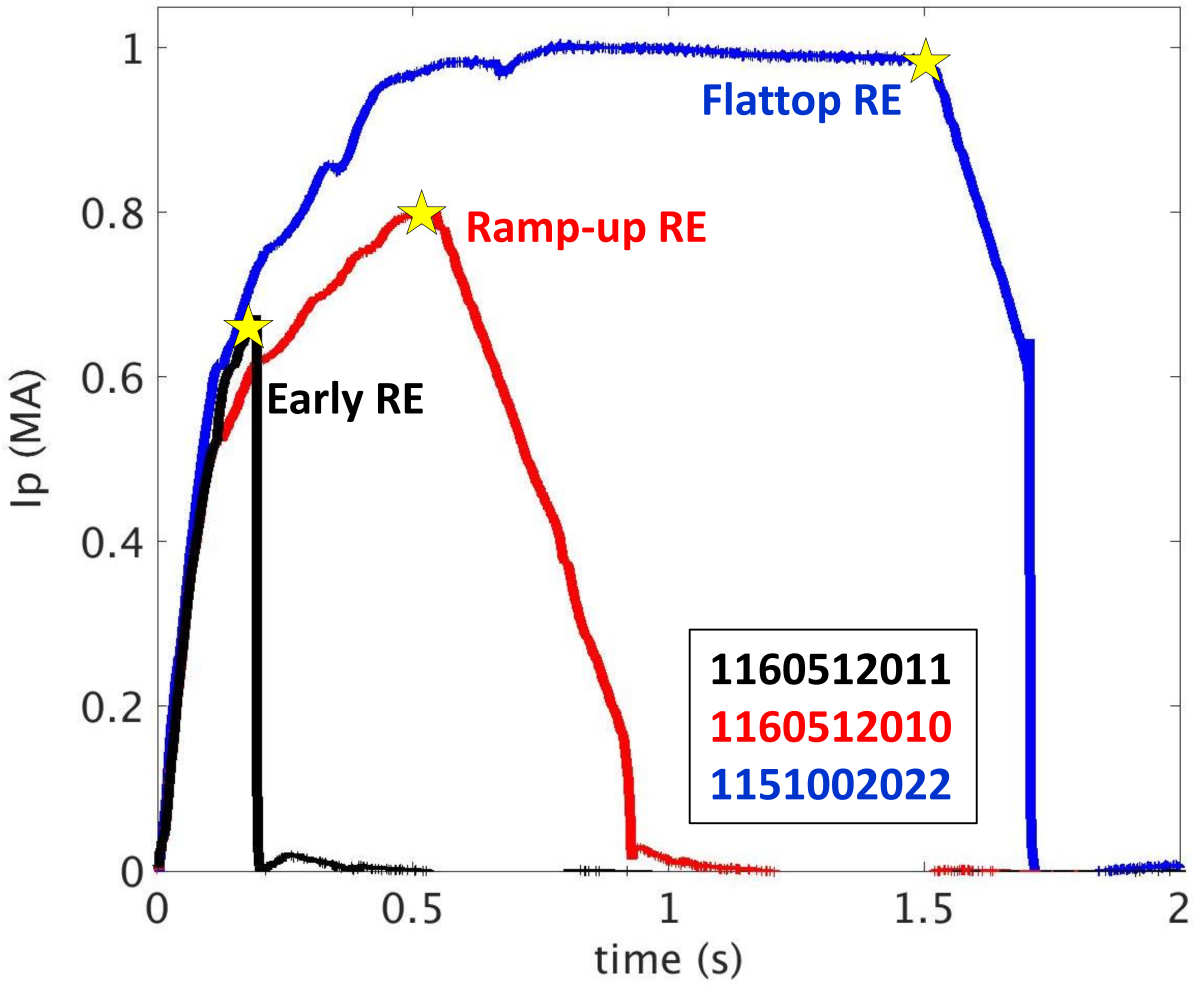
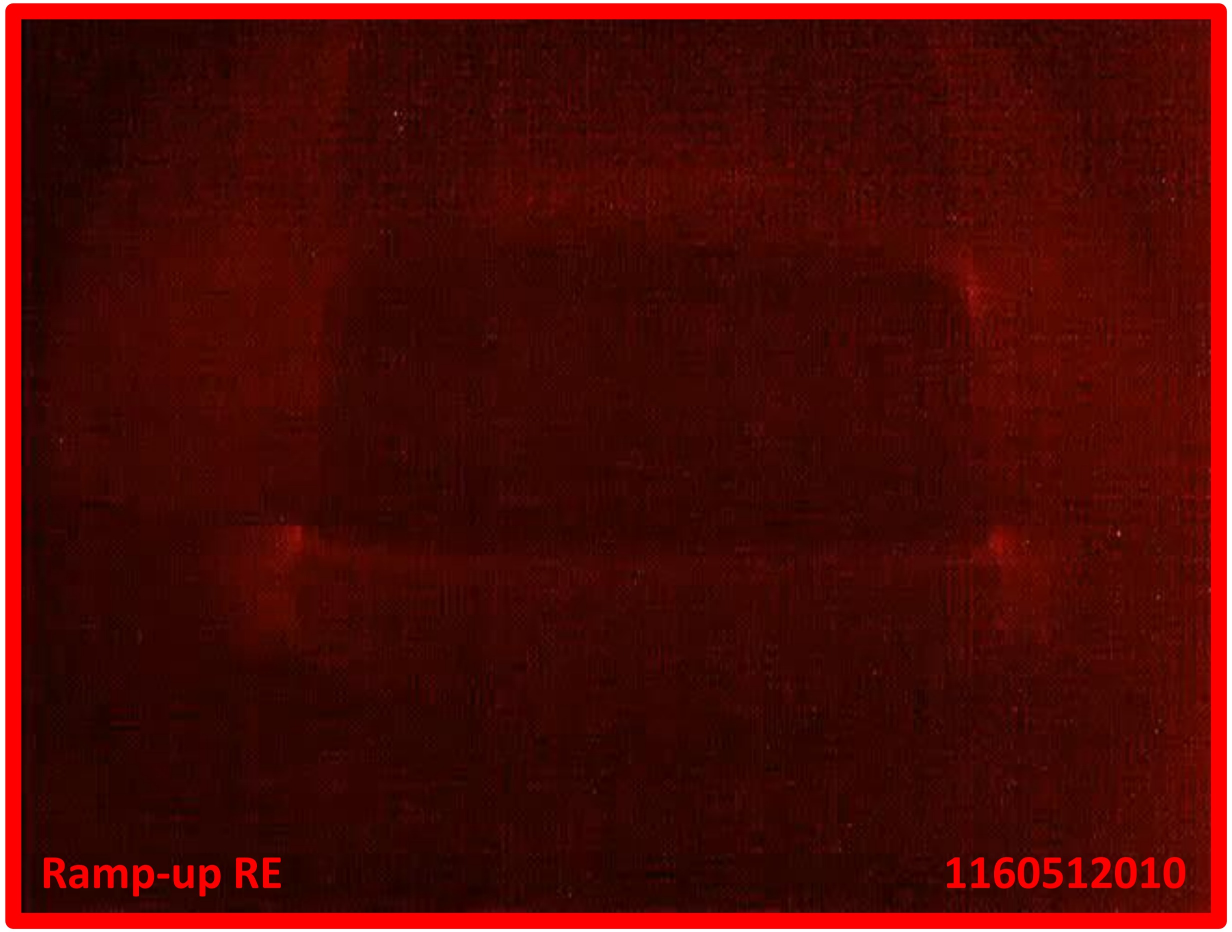




Toroidal cross section of C-Mod

Look at 3 different runaway shots





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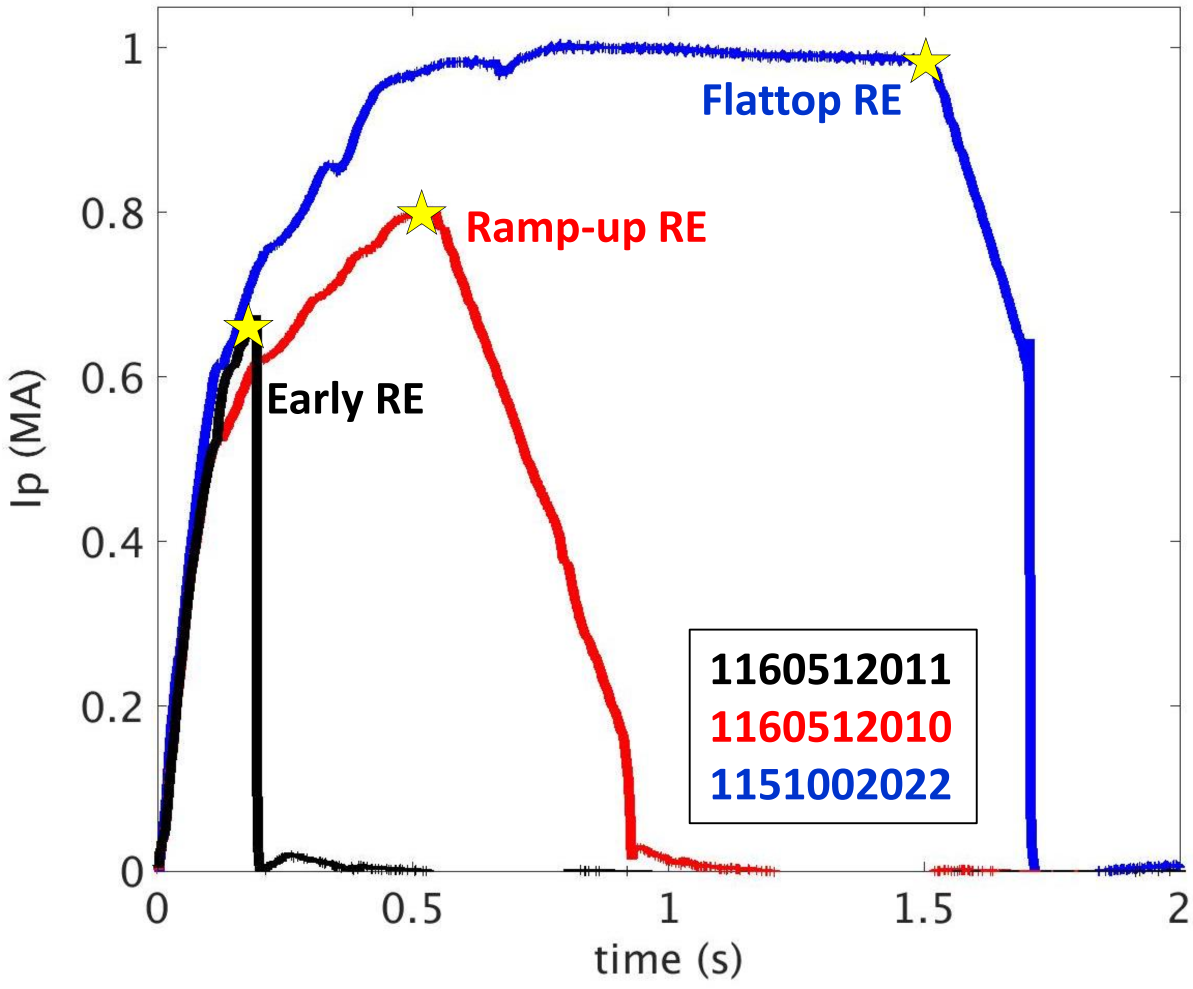

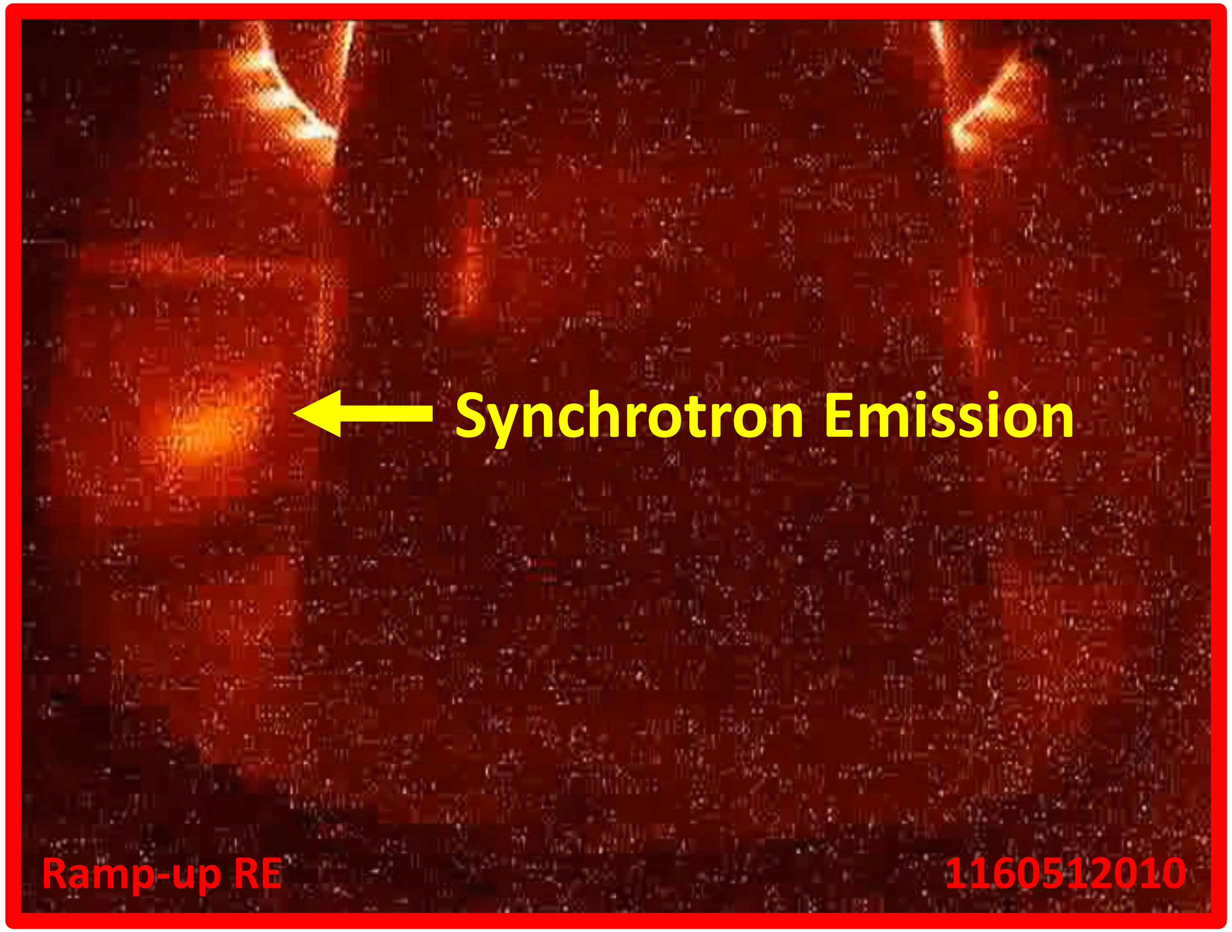

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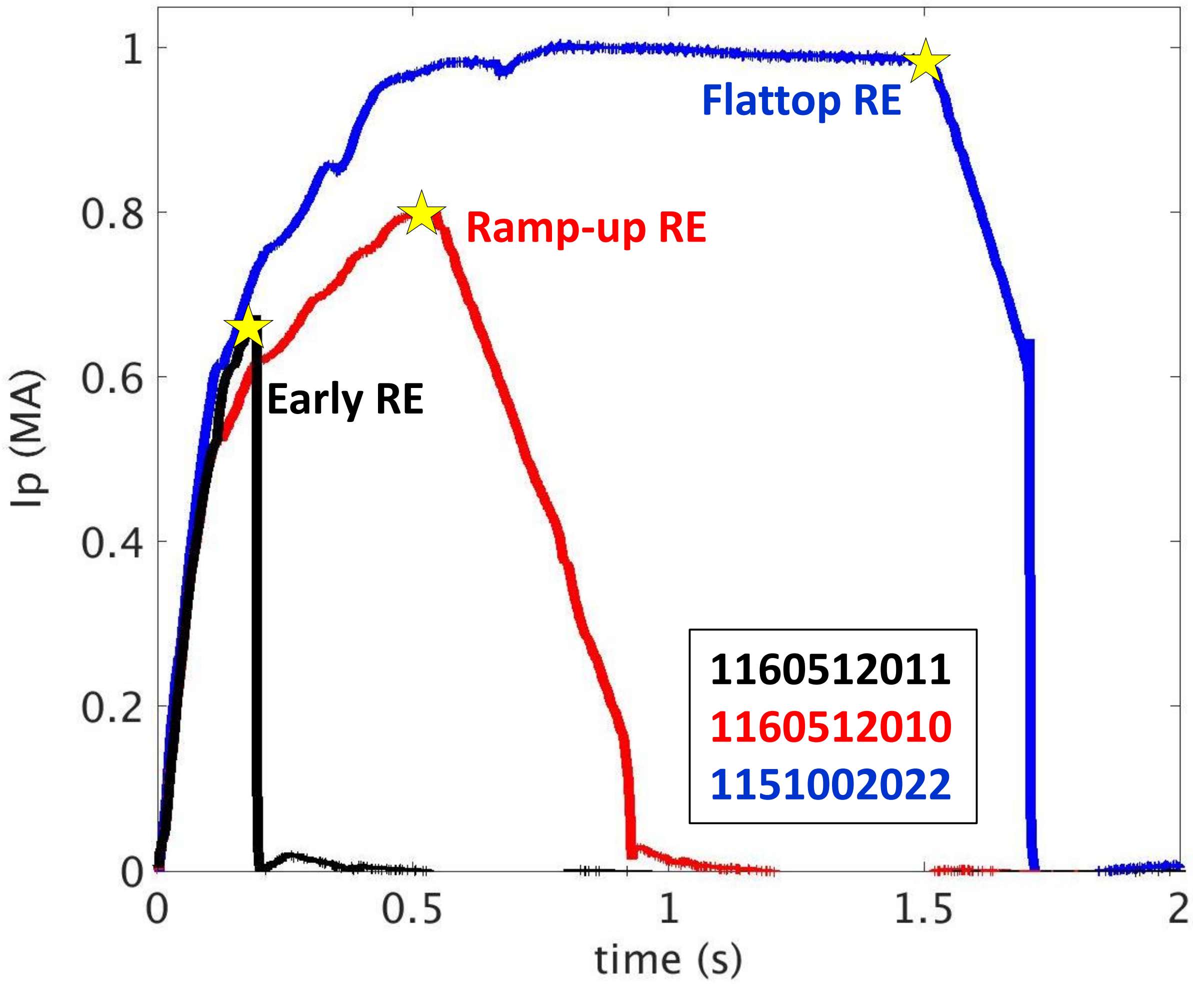
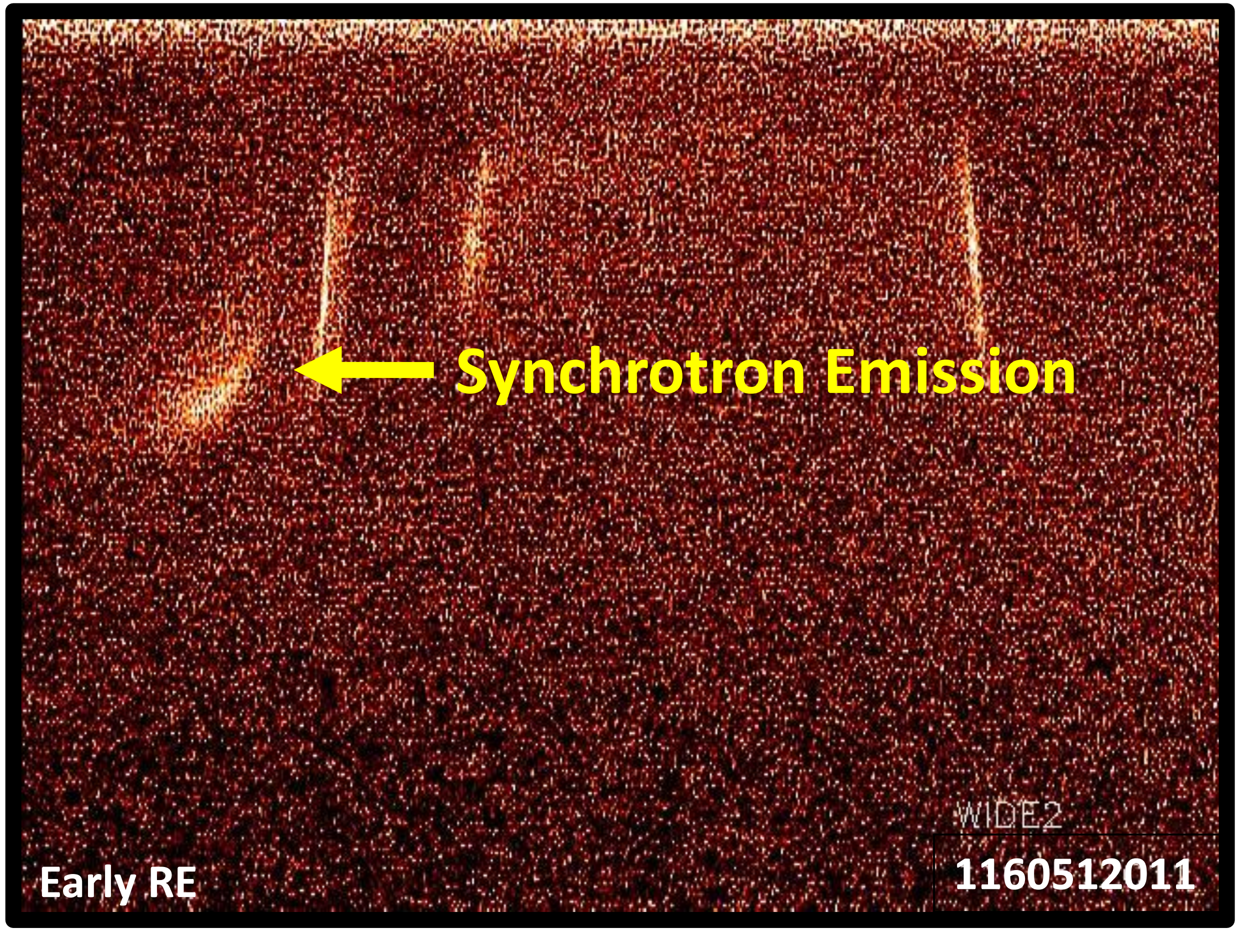
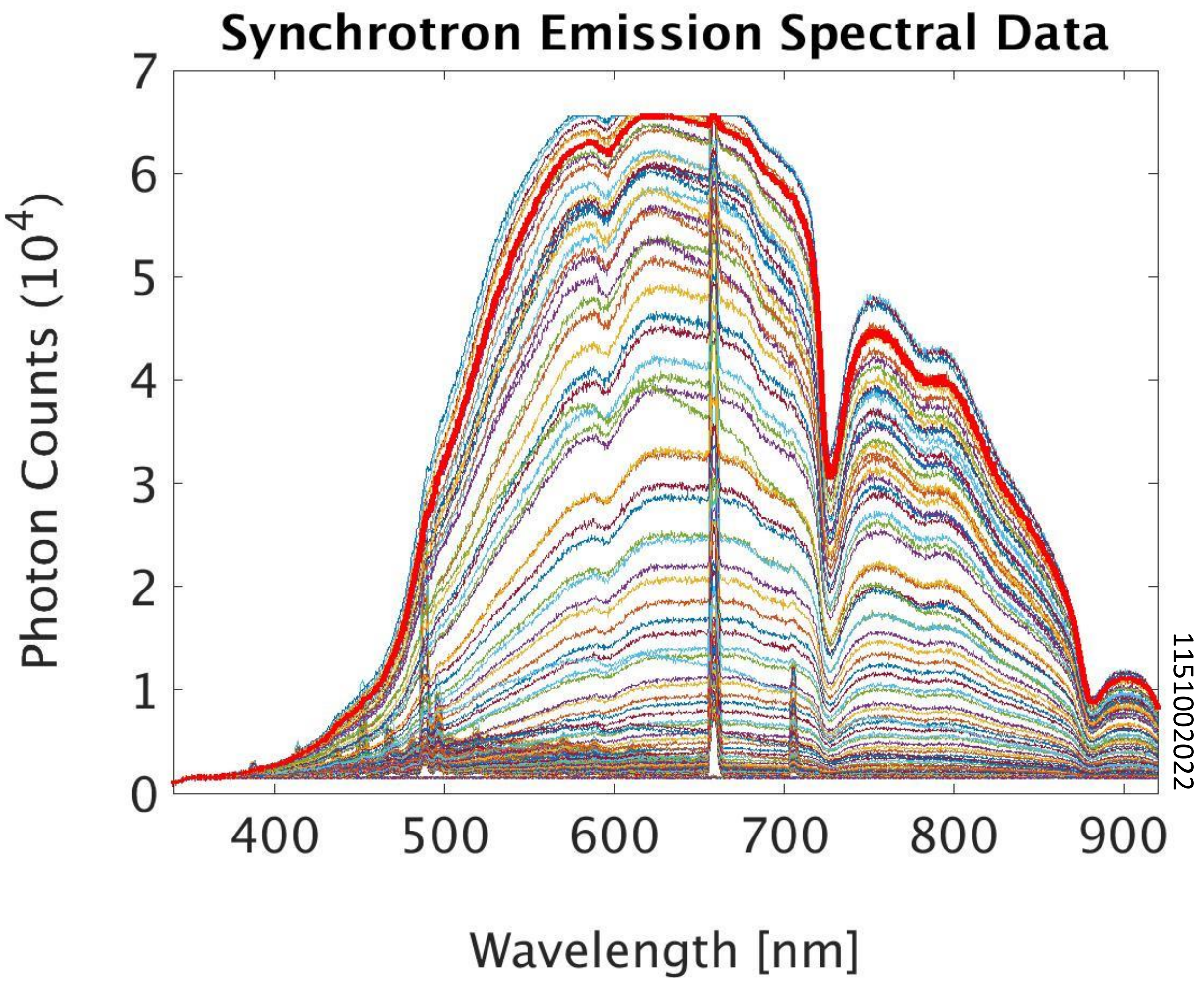




Flattop synchrotron emission data



Plasma parameters at **t = 1.5 s**:

- B_t = 5.35 T
- I_p ≈ 1 MA (end of flat-top)
- $\bar{n}_e = 2.5 \cdot 10^{19}$ m^{-3}
- T_{e0} = 4.25 keV
- a_{beam} ≈ 5 cm (as seen by camera)
- V_{loop} = 1.05 V

  → E = 0.25 V/m

  → **E/E_c = 12**

The red highlighted data is at t = 1.5 s and is used in this analysis.

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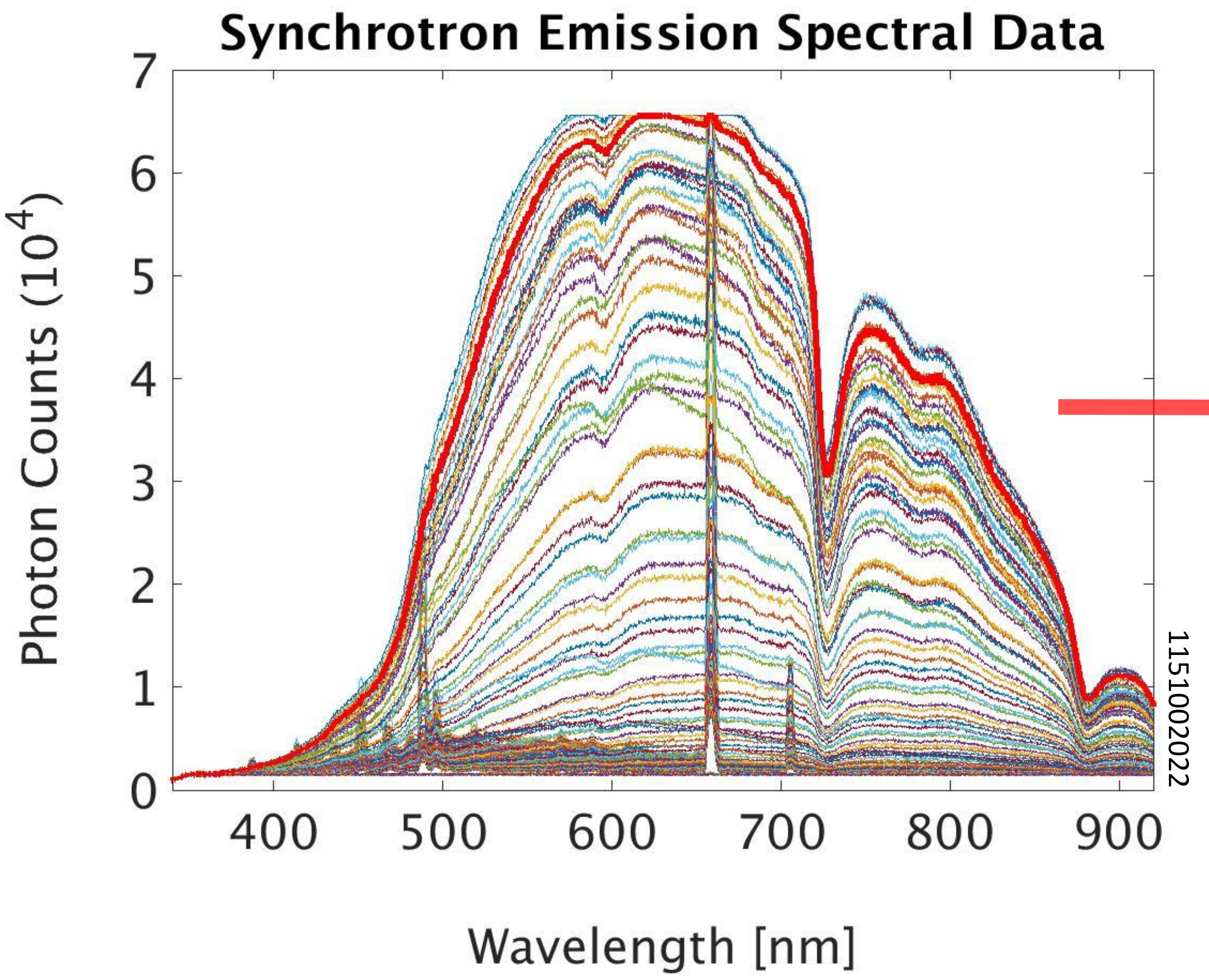

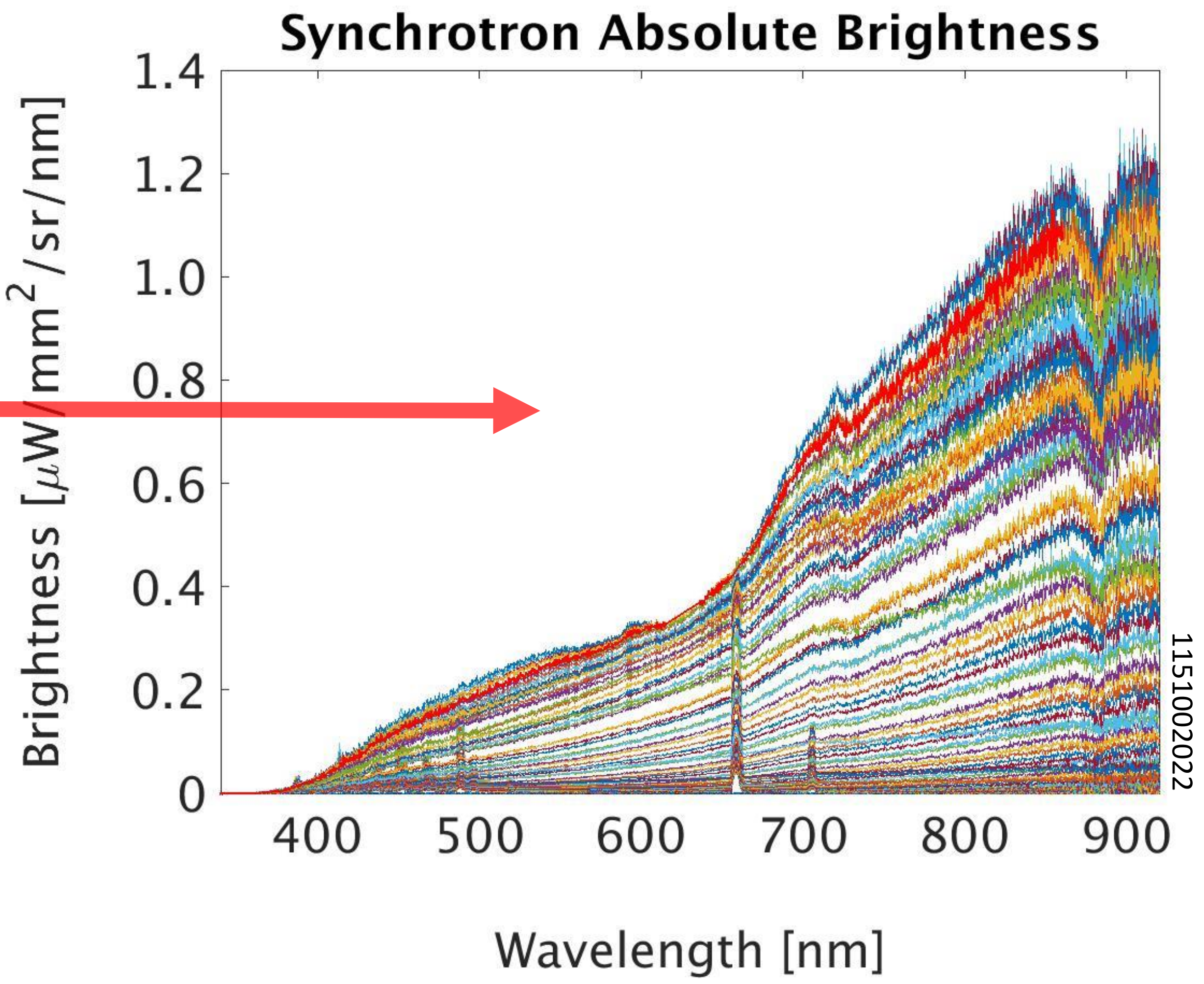

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Two models for the RE distribution

For a **mono-energetic** and mono-pitch RE beam, the brightness (W/m^3/sr) is [6]:

$$B_{mono}(\lambda, \boldsymbol{\theta}, \boldsymbol{p}) = \frac{2\ R\ \boldsymbol{n_r}}{\pi\ \theta_{eff}(\boldsymbol{p}, \boldsymbol{\theta})} P\left(\lambda, \theta_{eff}, \boldsymbol{p}\right)$$

where $\boldsymbol{n_r}$ is the density of REs emitting SR, $\boldsymbol{\theta} = \boldsymbol{v}_\perp/\boldsymbol{v}_\parallel = \boldsymbol{p}_\perp/\boldsymbol{p}_\parallel$ is the pitch, and $\boldsymbol{p} = \sqrt{\boldsymbol{E}^2/\boldsymbol{m}^2\boldsymbol{c}^4 - 1}$ is the normalized momentum.

[6] J.H. Yu, et al. Phys. Plasmas 20, 042113 (2013).

Alcator C-Mod

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For a **distribution**, f_{RE}, of energies and pitches [7],

$$f_{RE}(\boldsymbol{p_\parallel}, \boldsymbol{p_\perp}) = \frac{\boldsymbol{n_r}\, \hat{E}}{2\pi c_Z\, \boldsymbol{p_\parallel}\, ln\Lambda} \exp\left(-\frac{\boldsymbol{p_\parallel}}{c_Z ln\Lambda} - \frac{\hat{E}\, \boldsymbol{p_\perp^2}}{2\boldsymbol{p_\parallel}}\right),$$

the brightness (W/m^3/sr) is [4]:

$$B_{dist}(\lambda) = 4R \int\int \frac{1}{\theta_{eff}(\boldsymbol{p_\parallel}, \boldsymbol{p_\perp})} P(\lambda, \theta_{eff}, \boldsymbol{p}) f_{RE}(\boldsymbol{p_\parallel}, \boldsymbol{p_\perp}) \boldsymbol{p_\perp} dp_\parallel dp_\perp$$

[4] A. Stahl, et al. Phys. Plasmas 20, 093302 (2013). [6] J.H. Yu, et al. Phys. Plasmas 20, 042113 (2013). [7] T. Fülöp, et al. Phys. Plasmas 13, 062506 (2006).

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The RE density, $\boldsymbol{n_r}$, is estimated [4,6] using the plasma current carried by the REs , $\boldsymbol{I_r}$, and cross-sectional area, A_r, of the beam (as seen by our cameras):

$$\boldsymbol{n_r} = \boldsymbol{I_r}/(ecA_r)$$

During the discharge, we do not know $\boldsymbol{I_r}$, so we have to fit the data by varying the RE current for both the mono-energetic and continuum distributions.

[4] A. Stahl, et al. Phys. Plasmas 20, 093302 (2013). [6] J.H. Yu, et al. Phys. Plasmas 20, 042113 (2013). [7] T. Fülöp, et al. Phys. Plasmas 13, 062506 (2006).

Mono-energetic fit matches flattop data

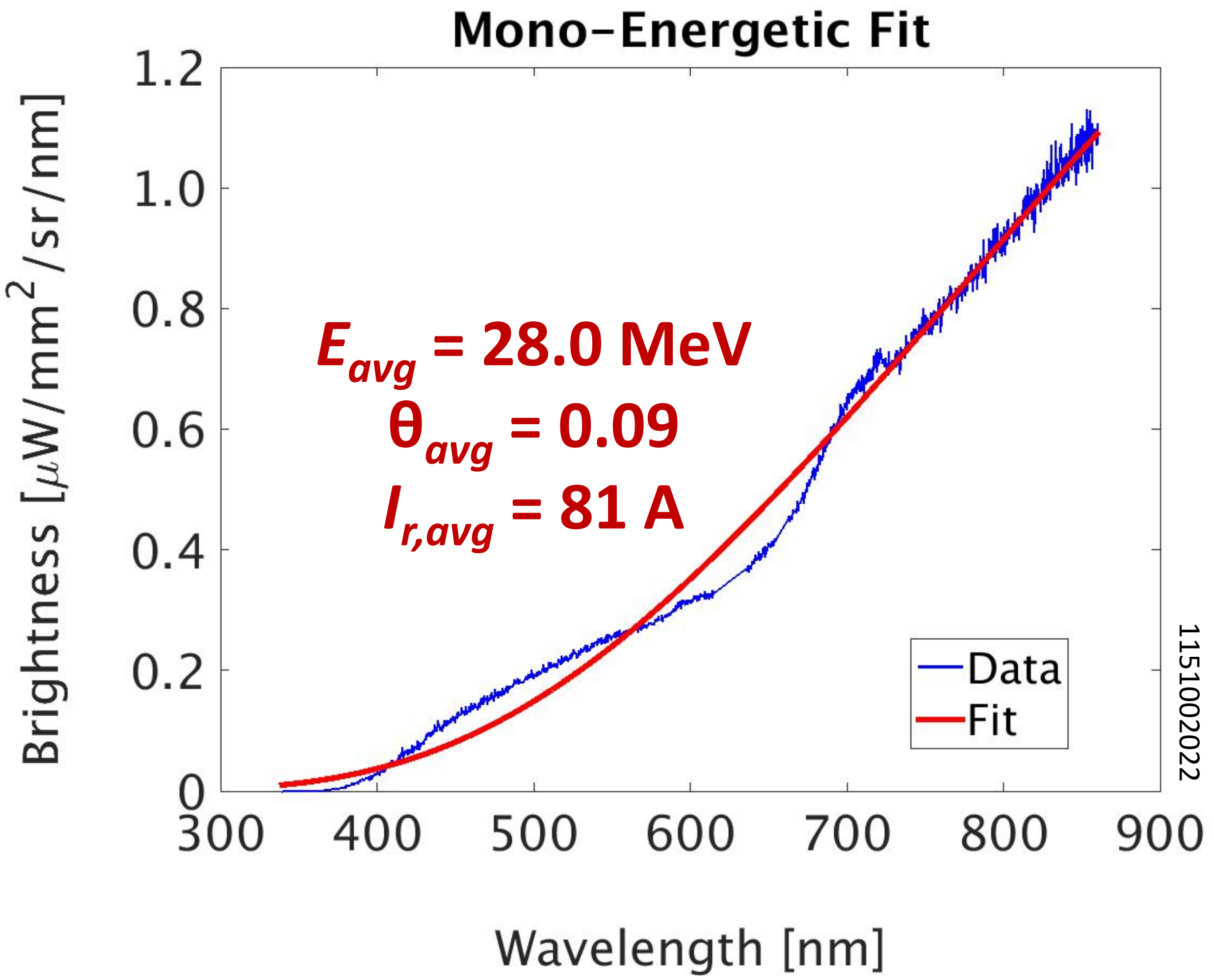
- This data can be well-fit for a range of RE energies, pitches, and currents:

$$24.8\ \text{MeV} \le E_{mono} \le 30.6\ \text{MeV}$$

$$0.070 \le \theta = \frac{v_\perp}{v_\parallel} \le 0.125$$

$$77\ \text{A} \le I_{r,mono} \le 82\ \text{A}$$

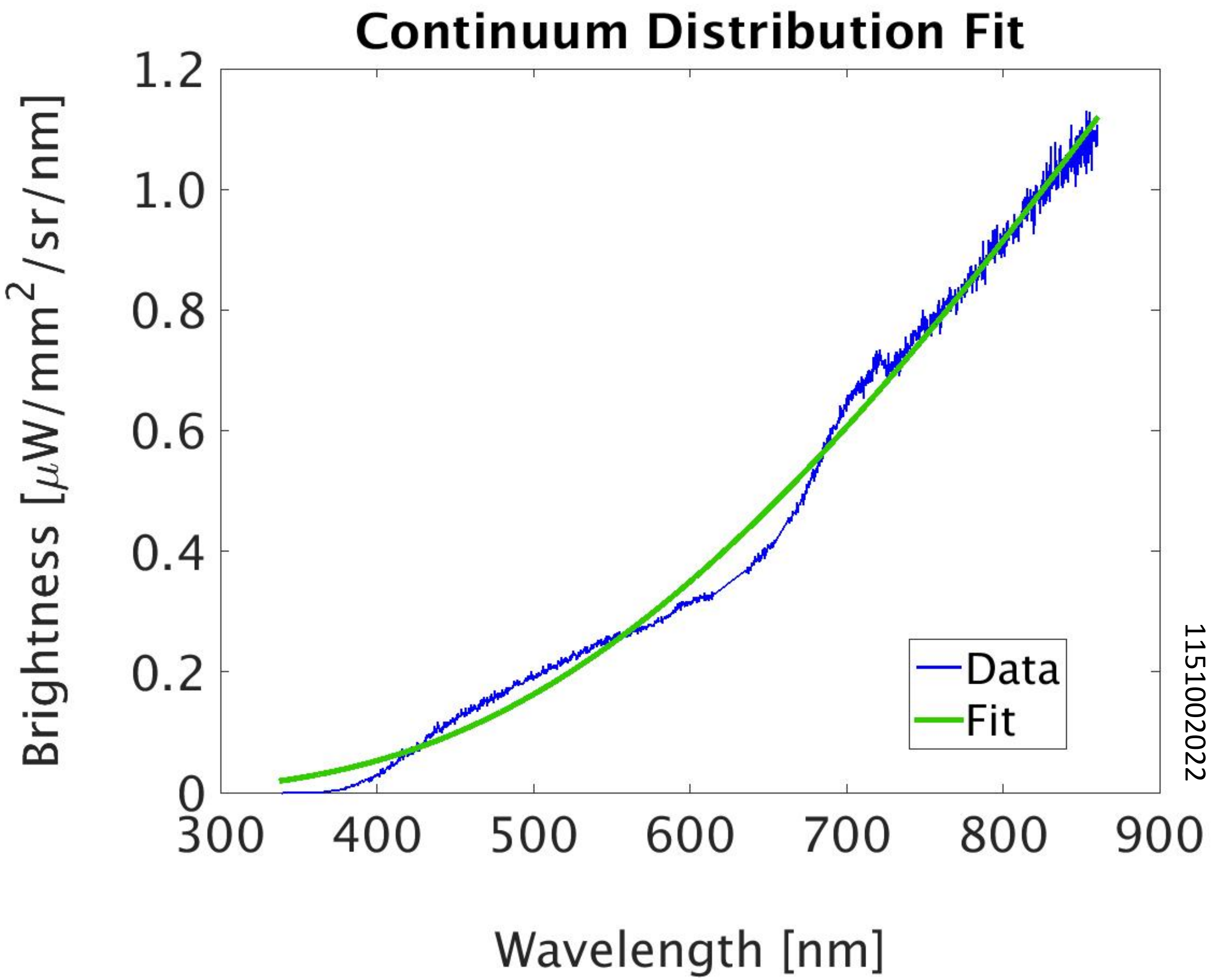
- Assuming all REs emit SR at 28 MeV and pitch of 0.09, this means they only carry ~100 A of the 1 MA plasma current.



Continuum distribution matches flattop data

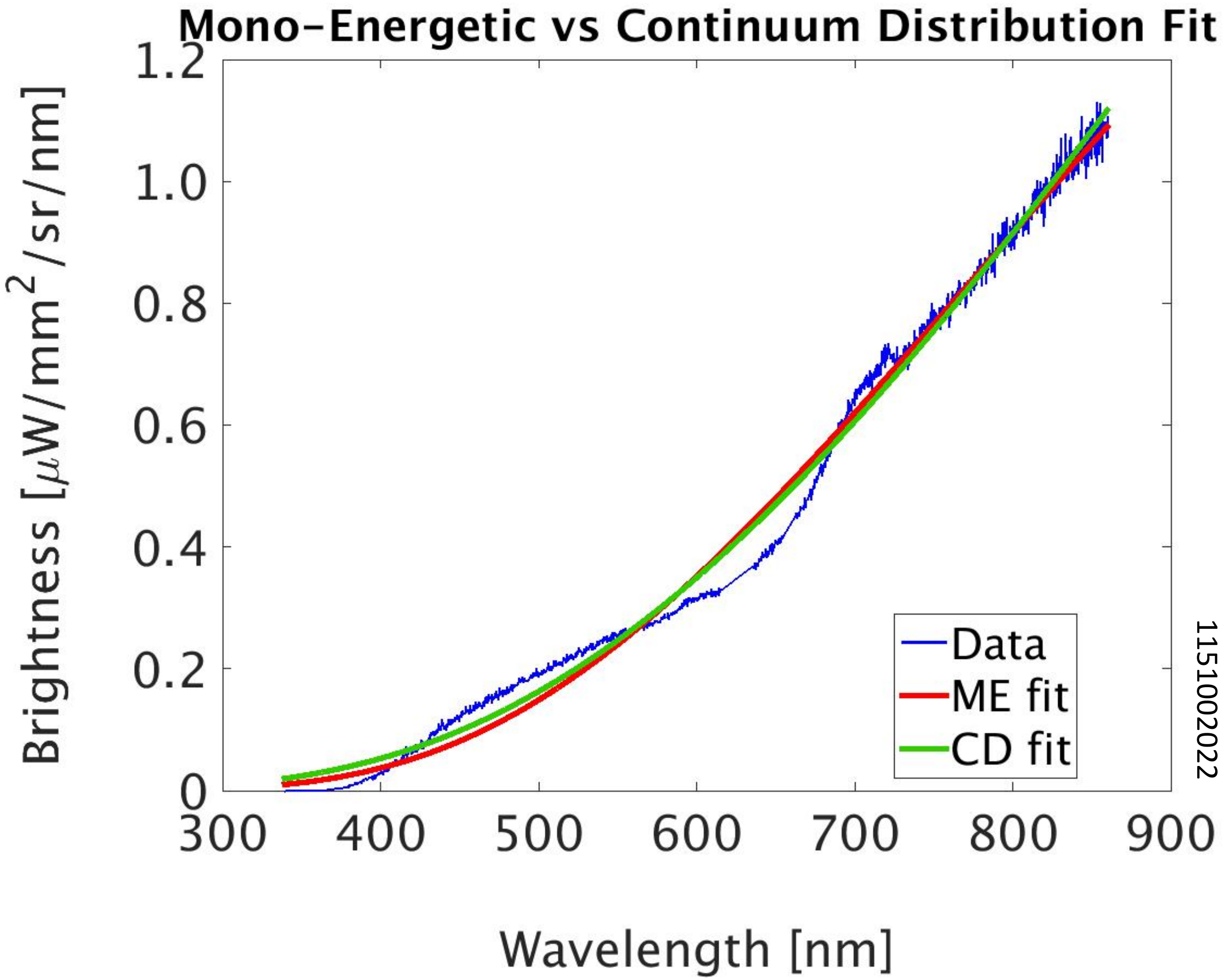
This best fit calculates:

- $E_{max,dist}$ = 19.7 MeV
  - About 10 MeV less than E_{mono}
- $I_{r,dist}$ = 3.5 kA
  - Accounts for <1% of the total plasma current, but more than $I_{r,mono}$
- $Z_{eff,dist}$ = 3
  - Lower bound of fitting range (3 – 7).

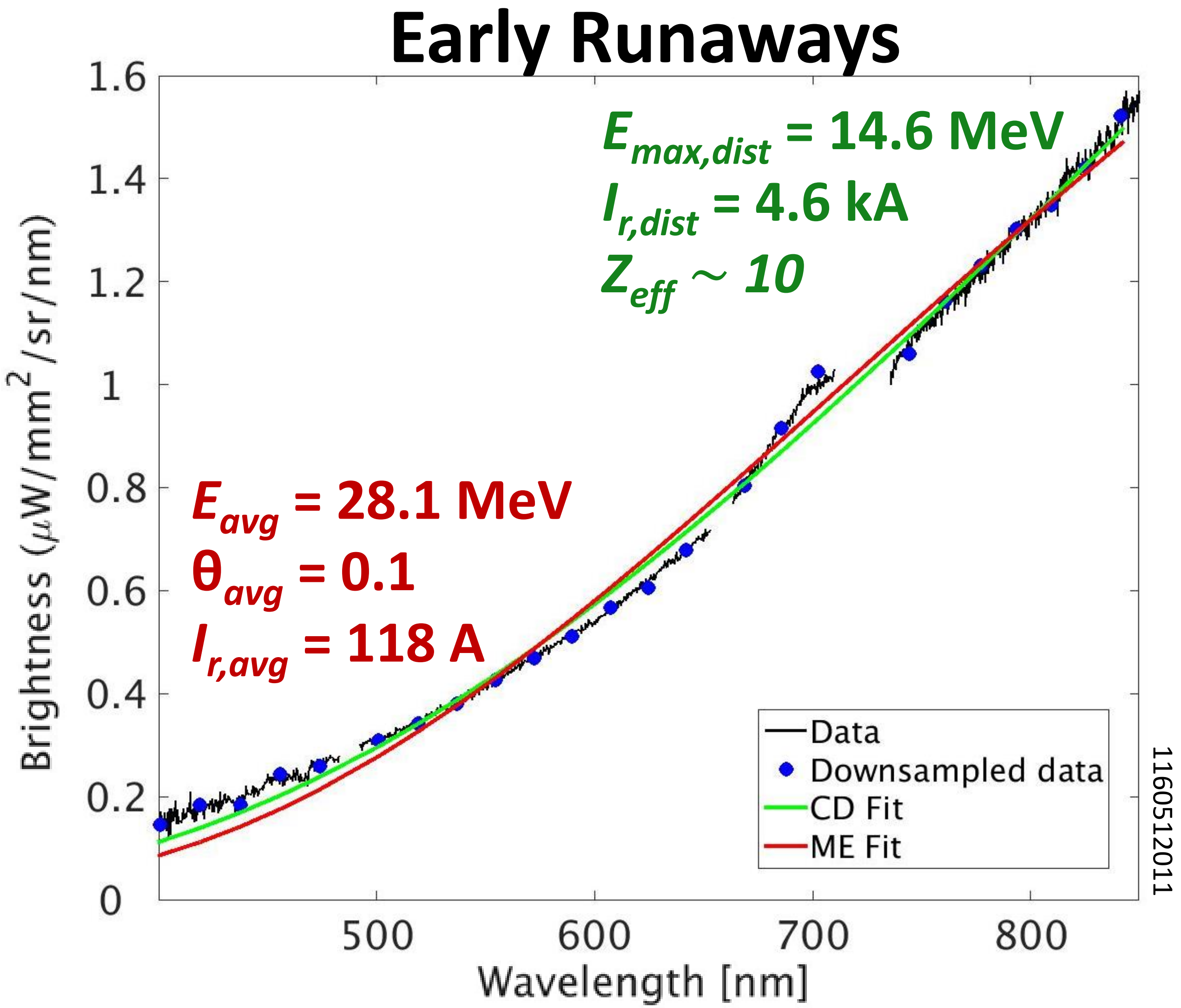


Both flattop fits are comparable

The **mono-energetic** and **continuum distribution** fits are very similar, with about the same goodness of fit.



Both ME and CD fits are again comparable



Plasma parameters at t = 0.18 s:

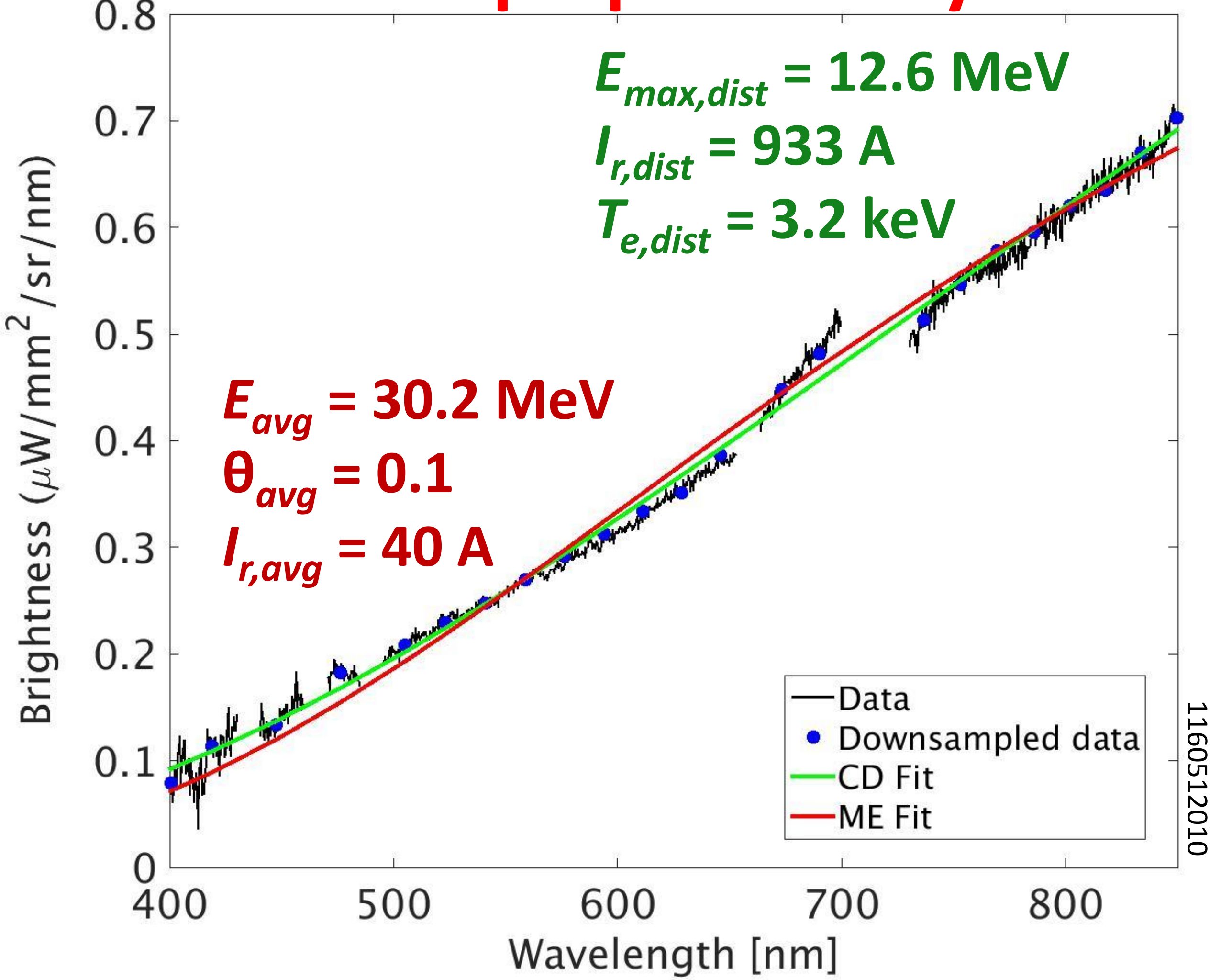
- B_t = 5.24 T
- **I_p ≈ 670 kA**
- $\bar{n}_e = 5.9 \cdot 10^{19}$ m^{-3}
- T_{e0} = 2.5 keV
- a_{beam} ≈ 6 cm (as seen by camera)
- V_{loop} ≈ 2.3 V

→ E = 0.54 V/m

→ **$E/E_c \approx 11$**

Both ME and CD fits are again comparable

Ramp-up Runaways



Plasma parameters at t = 0.54 s:

- B_t = 5.36 T
- **I_p ≈ 800 kA**
- $\bar{n}_e = 6.6 \cdot 10^{19}$ m^{-3}
- T_{e0} = 2.3 – 3.2 keV
- a_{beam} ≈ 7 cm (as seen by camera)
- V_{loop} ≈ 1.1 V

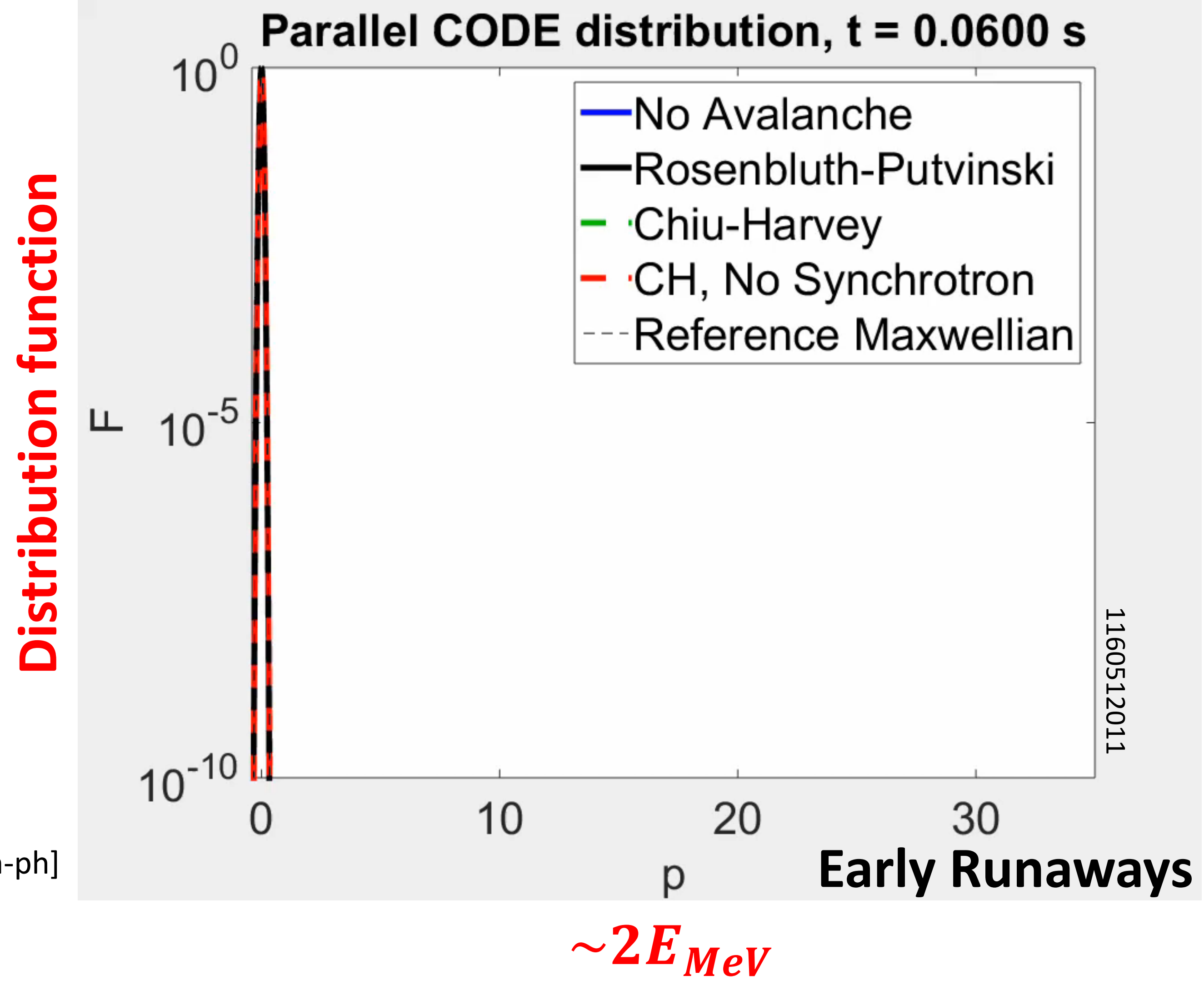
  → E = 0.26 V/m

  → **E/E_c ≈ 4.8**

Use CODE [5,9] to solve the forward problem

Alcator C-Mod

- Time dependent parameters:
  - $T_{e0}(t)$
  - $\bar{n}_e(t)$
  - $V_{loop,0}(t) \rightarrow E(t)$
  - $Z_{eff}(t)$
  - B → Synchrotron
- Secondary avalanching source:
  - Rosenbluth-Putvinskii (**RP**) [10]
  - Chiu-Harvey (**CH**) [11,12]



[5] M. Landreman, et al. Computer Physics Communications 185, 847 (2014).
[9] A. Stahl, et al., to appear in Nucl. Fusion. arXiv:1601.00898 [physics.plasm-ph]
[10] M. N. Rosenbluth, S.V. Putvinskii. Nucl. Fusion 37, 10 (1997).
[11] S. C. Chiu, et al. Nucl. Fusion 38, 1711 (1998).
[12] R. W. Harvey, et al. Phys. Plasmas 7, 4590 (2000).

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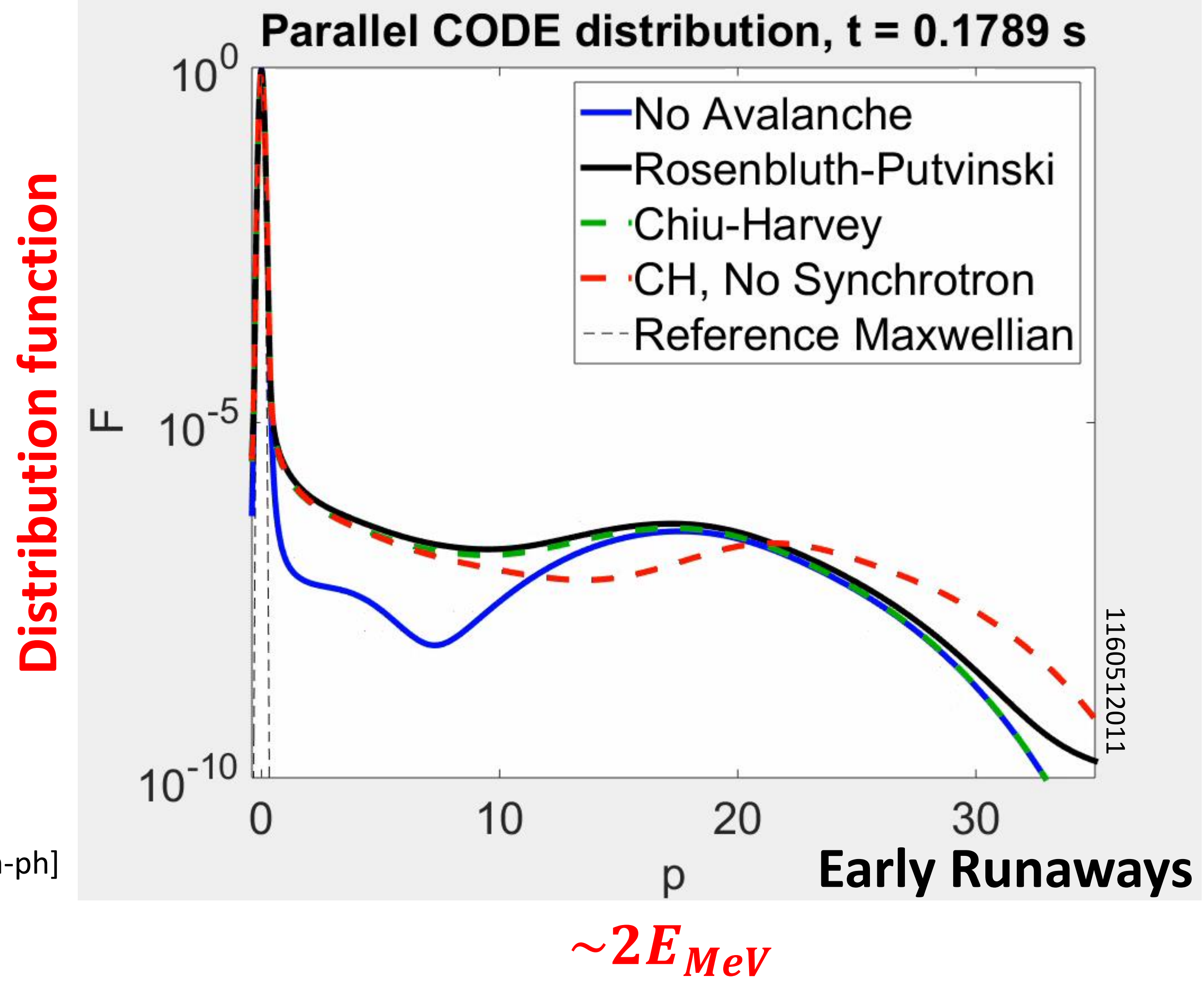

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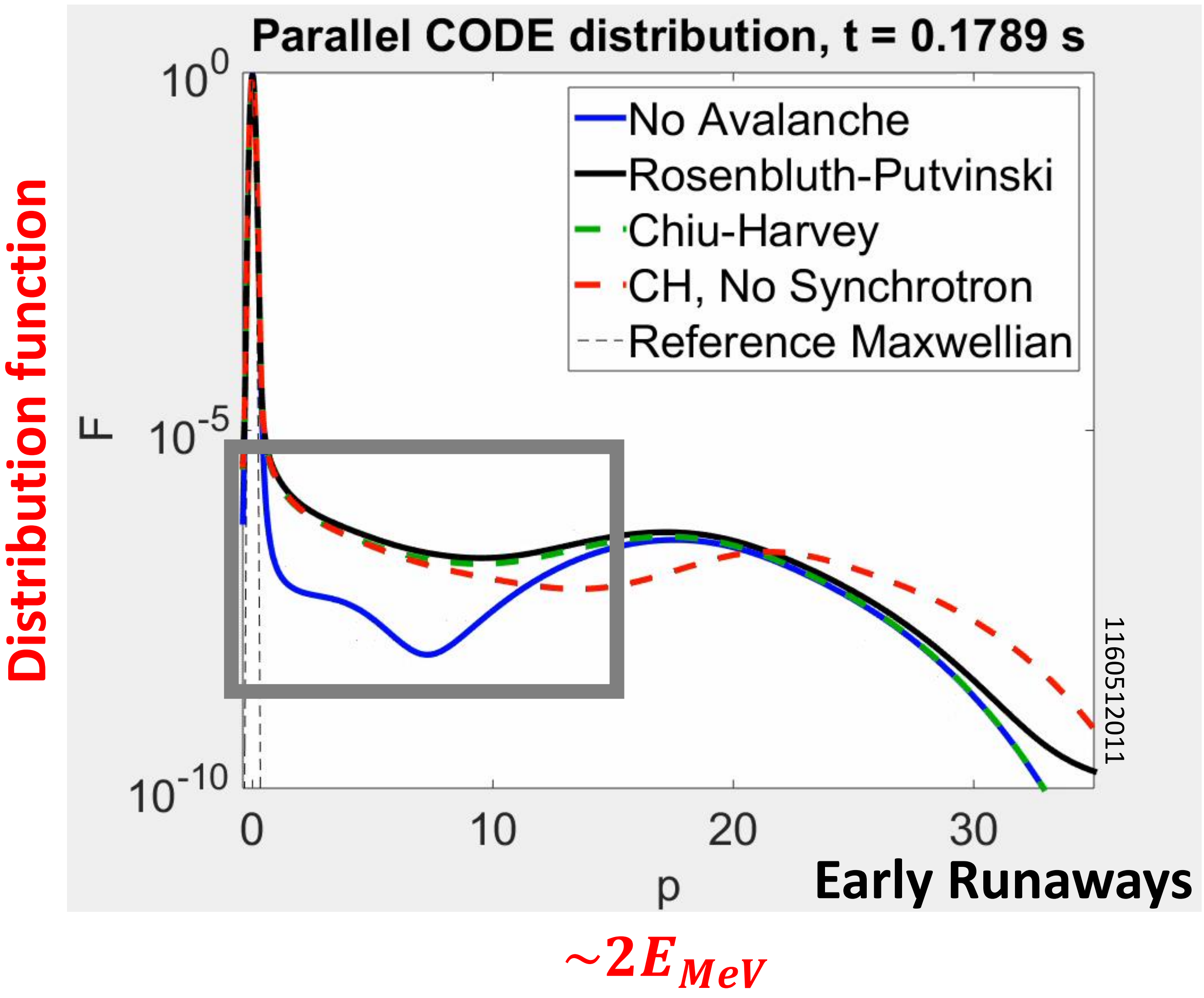
Bump forms on tail of **Early Runaway** distribution

- Avalanche populates lower energies



Bump forms on tail of **Early Runaway** distribution

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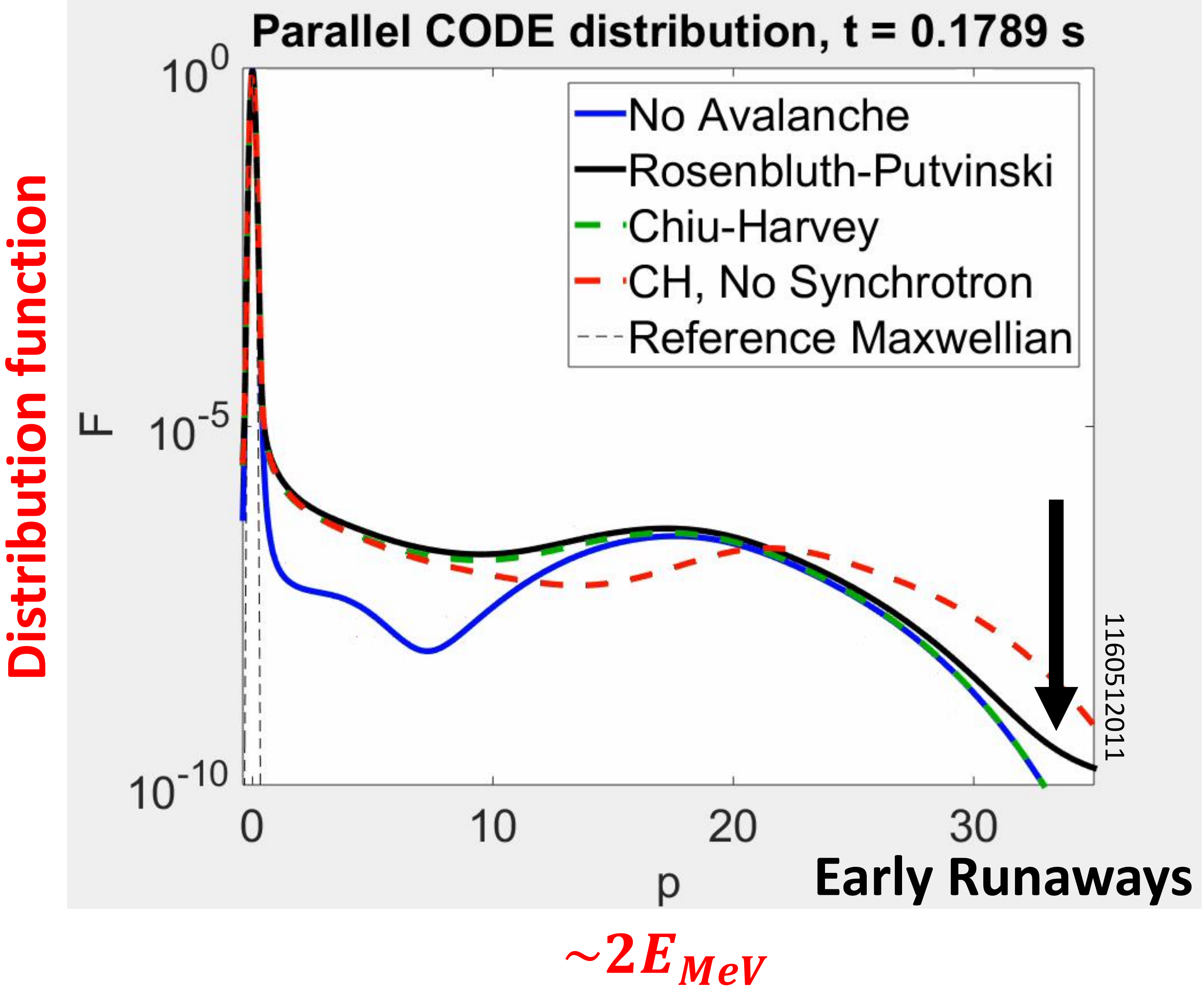

Bump forms on tail of **Early Runaway** distribution

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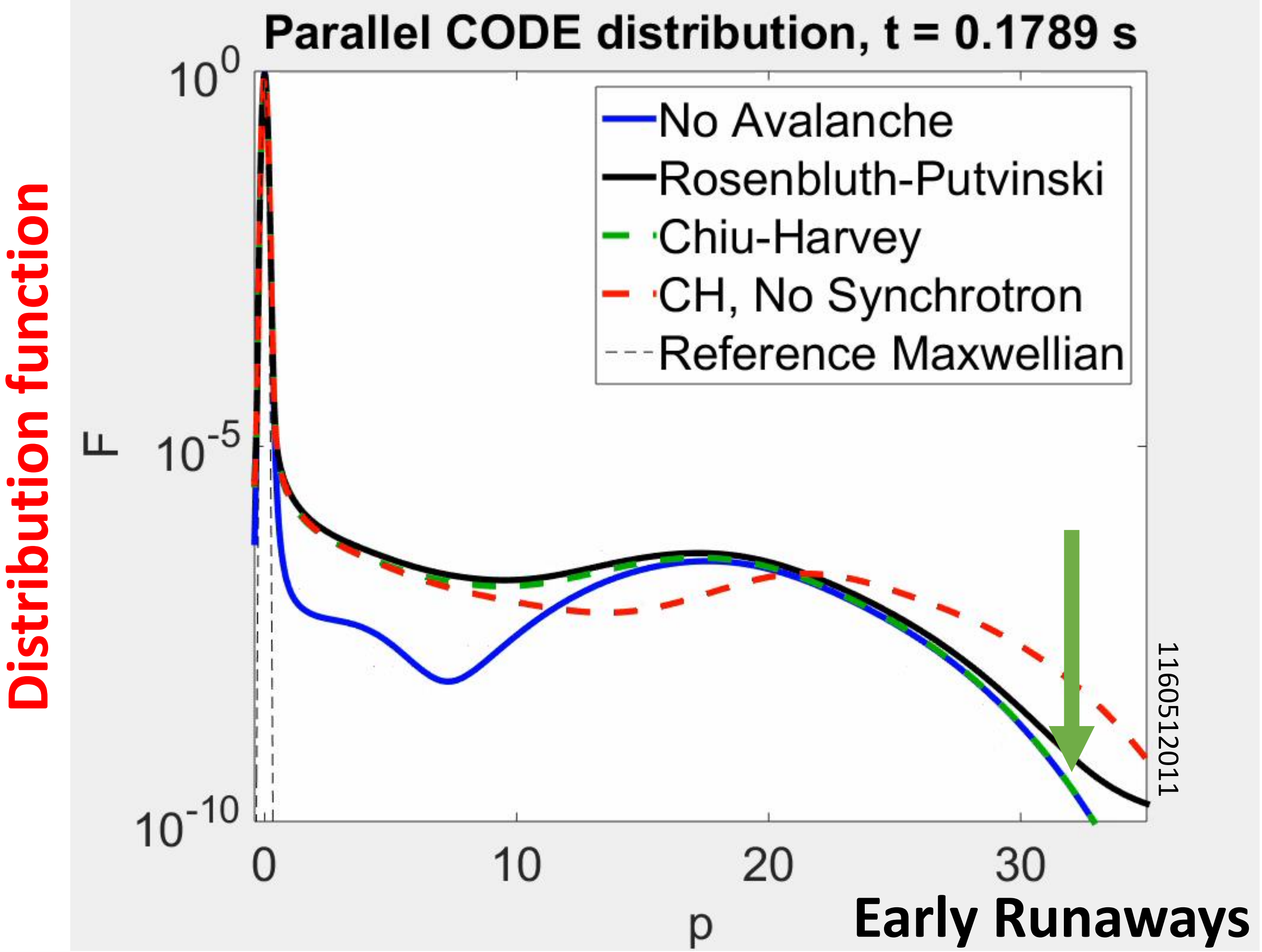
- **RP Avalanche** extends tail
- **CH Avalanche** matches **No Avalanche** case at high energies

→Primary (Dreicer [13]) generation dominates



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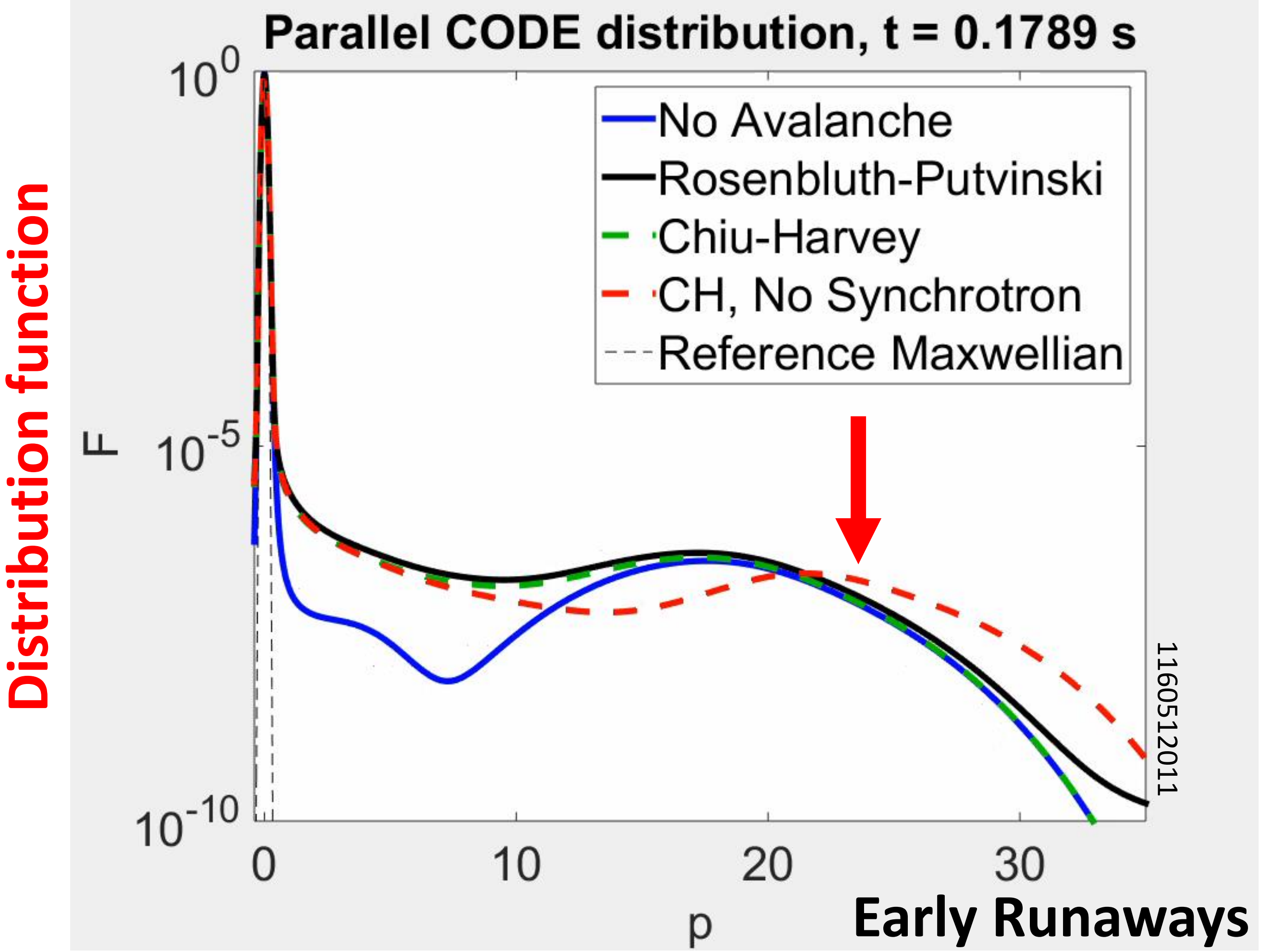
- **No Synchrotron** case still forms bump

→Dynamic plasma parameters can form bump



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Alcator C-Mod

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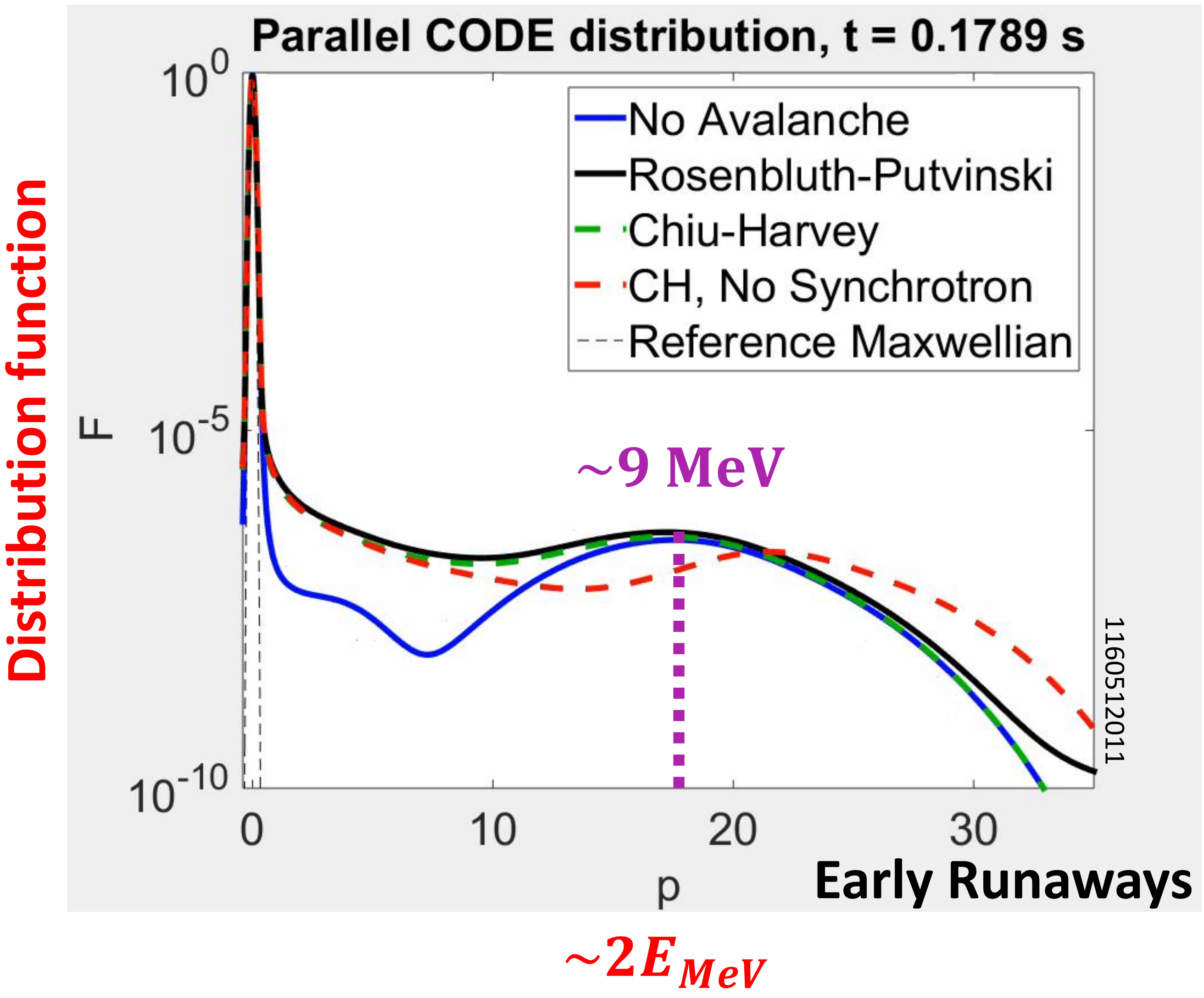
- **No Synchrotron** case still forms bump

→Dynamic plasma parameters can form bump

→**Synchrotron limits bump energy**



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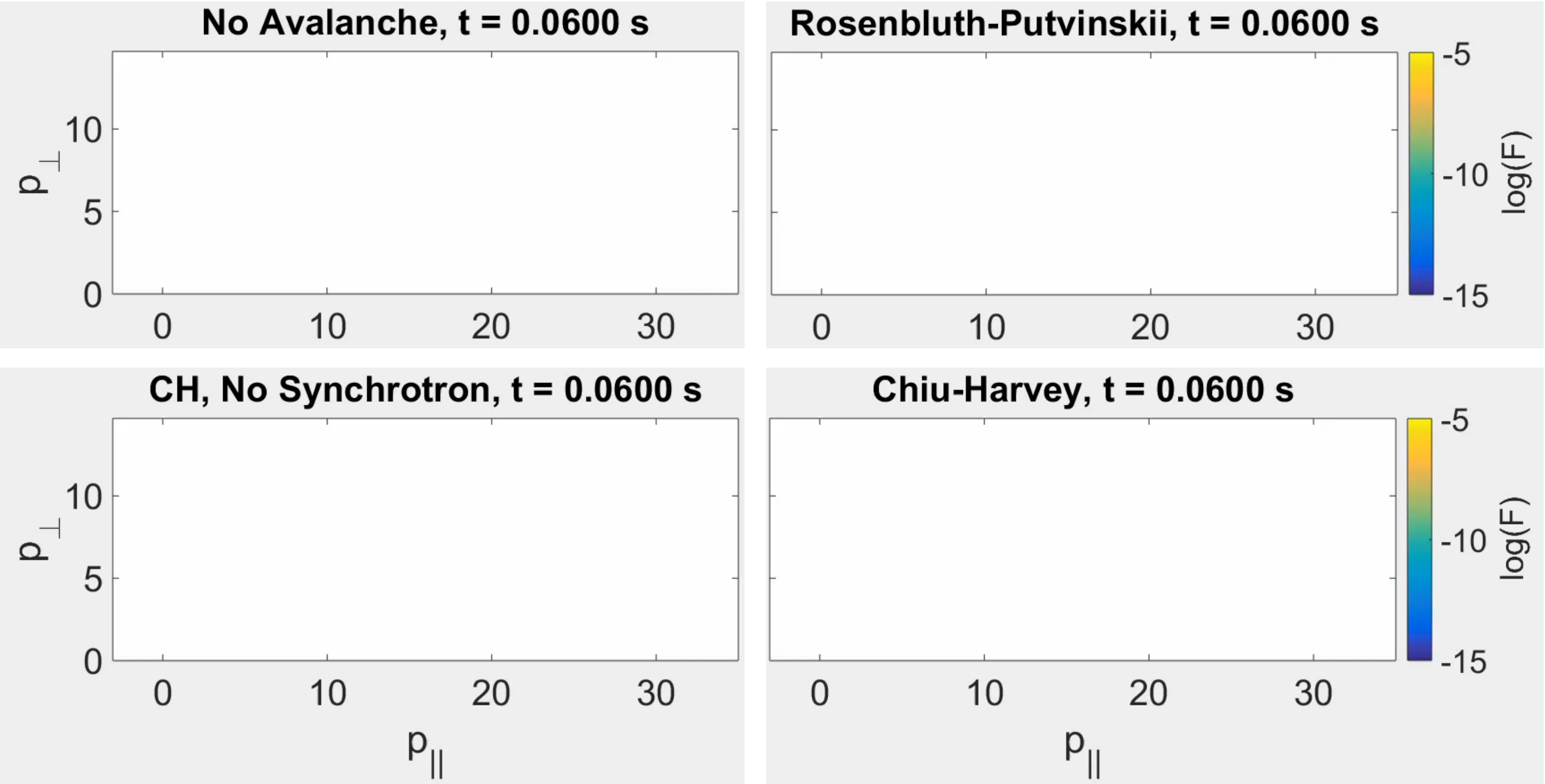
Synchrotron power loss more easily seen in 2D



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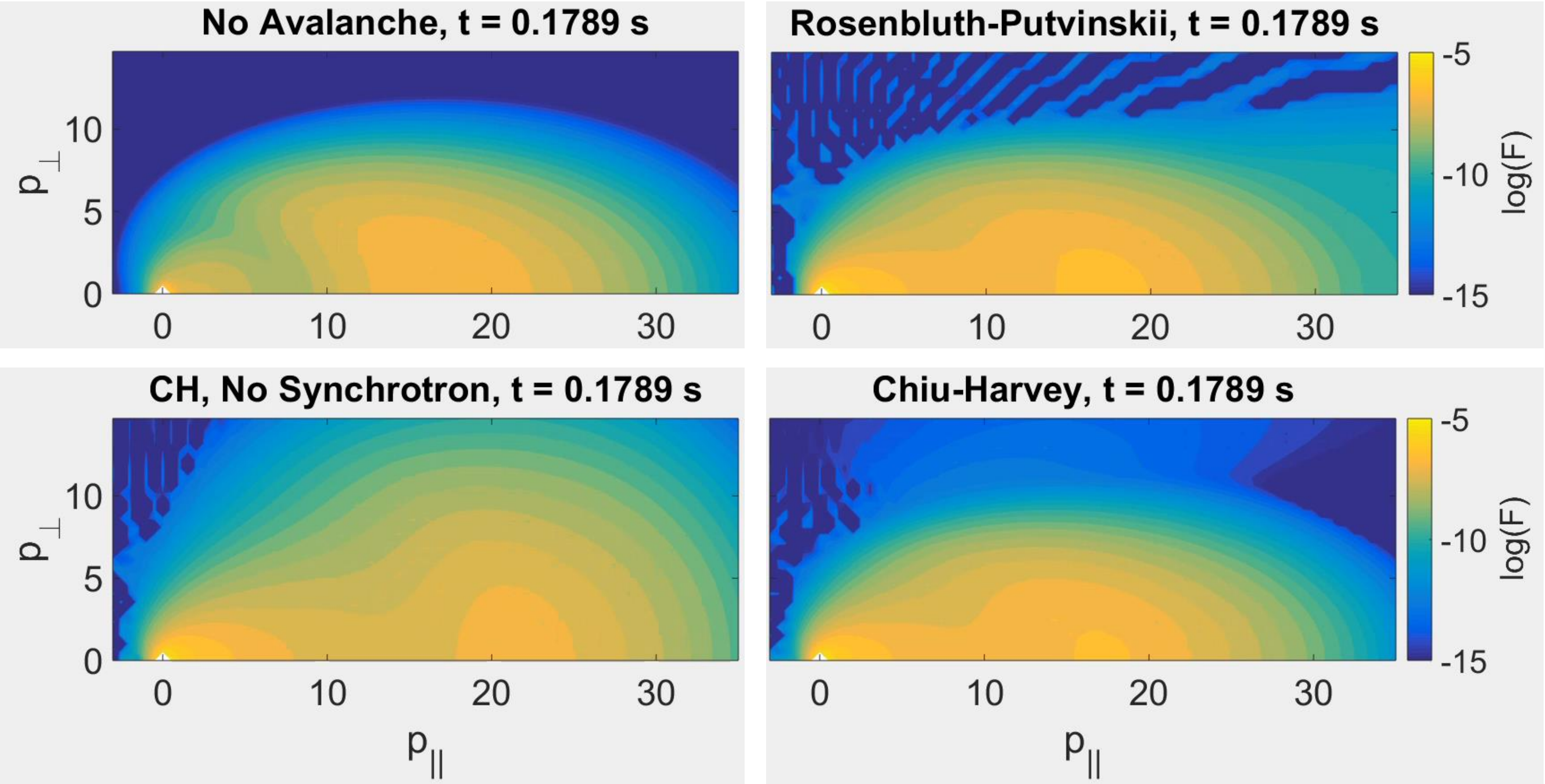

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Summary and future work

Mono-energetic and **continuum distribution** calculations both fit C-Mod experimental data equally well.

A time-dependent CODE model of one C-Mod runaway discharge calculates a **bump on the tail** of the energy distribution function which is:

- Dominated by **primary generation**
- Limited by **synchrotron radiation**
- Formed by **dynamic plasma parameters**

Next steps:

- Calculate the **synchrotron brightness** from CODE's distribution functions and **compare to experiment**
- Run CODE for the other runaway discharges
- Use a **non-linear** solver for discharges with runaway fractions > 10-15% (*see Adam Stahl's presentation Wednesday, 9:30am*)

References

[1] P. Aleynikov, et al. Phys. Rev. Lett. 114, 155001 (2015).

[2] J. Decker, et al. Plasma Phys. Contr. Fusion 58, 025016 (2016).

[3] E. Hirvijoki, et al. J. Plasma Phys., vol. 81, 47810502 (2015).

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[9] A. Stahl, et al. "Kinetic modelling of runaway electrons in dynamic scenarios," to appear in Nucl. Fusion. arXiv:1601.00898 [physics.plasm-ph]

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Backup slides

Runaway electrons in C-Mod

Alcator C-Mod

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$\bar{n}_e = 0.2 - 2 \cdot 10^{20}\ m^{-3}$

$T_{e0} = 1 - 5$ keV

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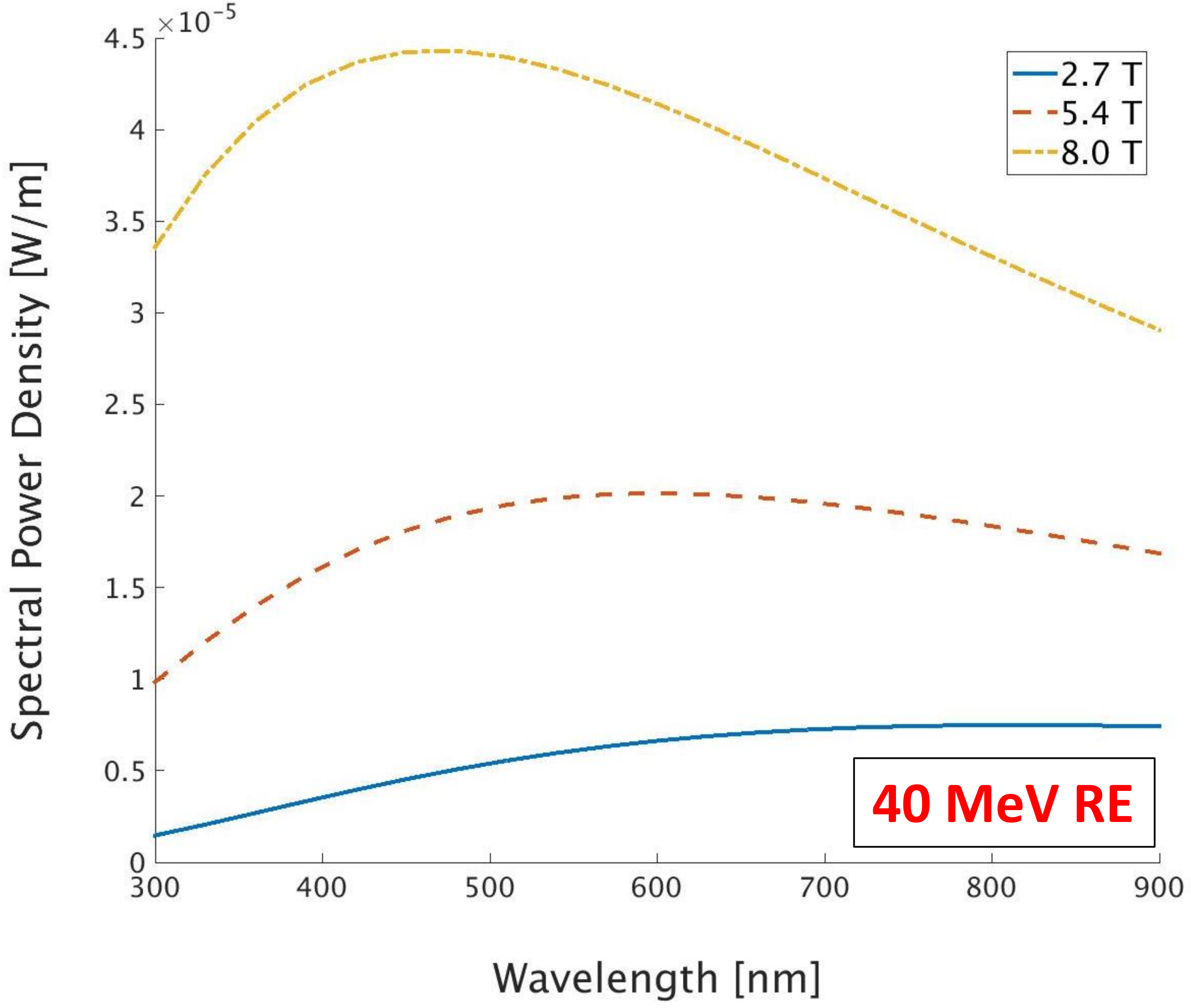

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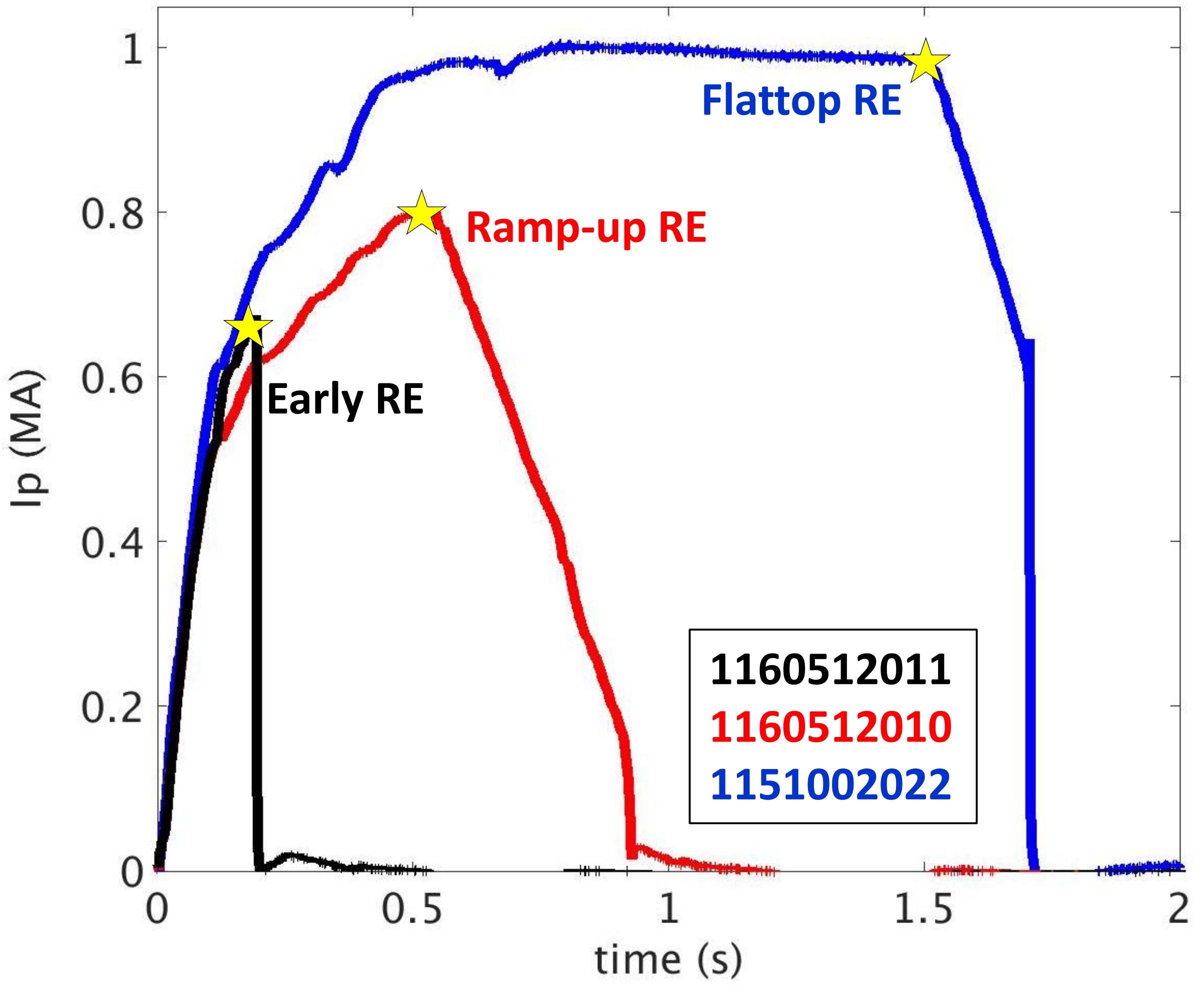

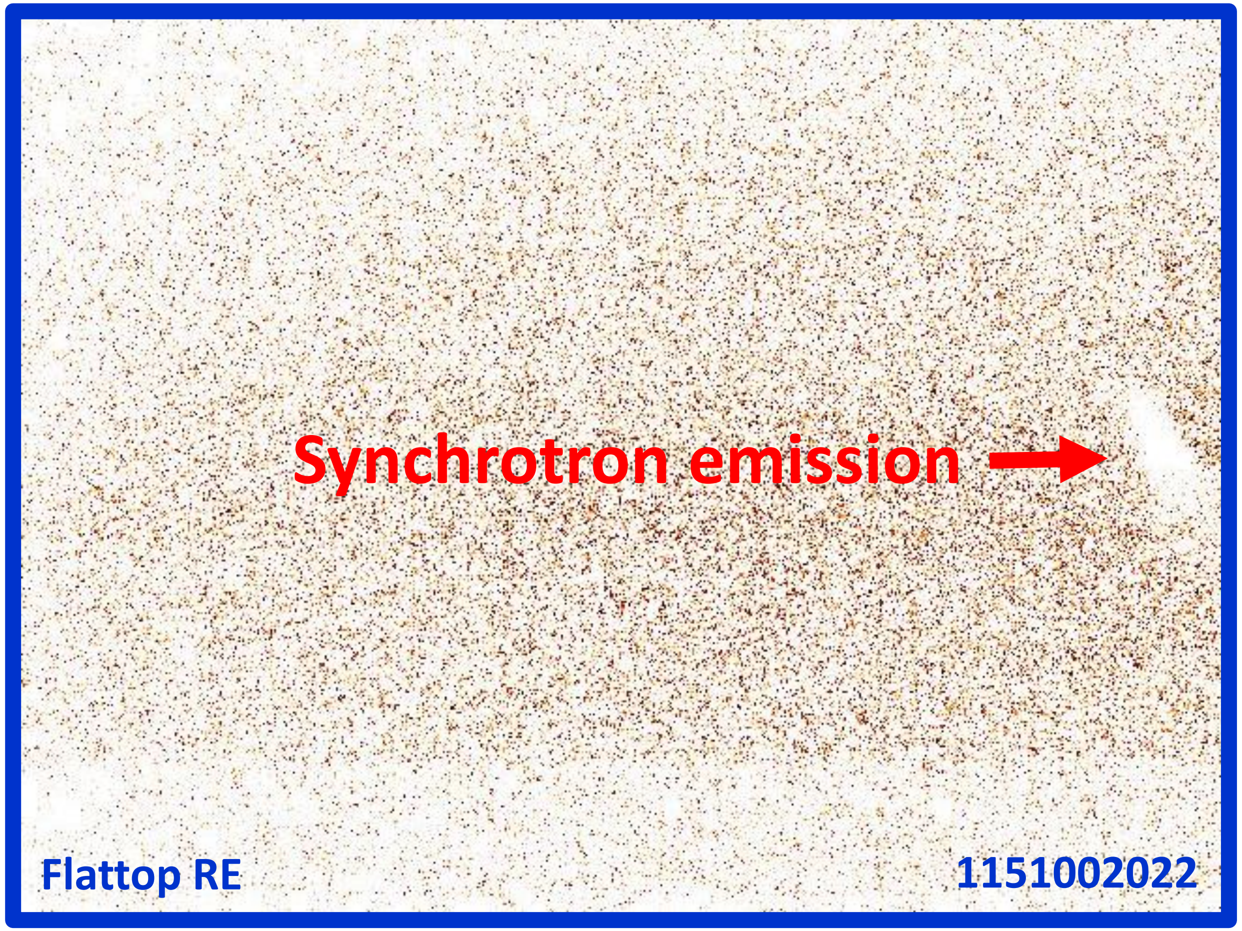

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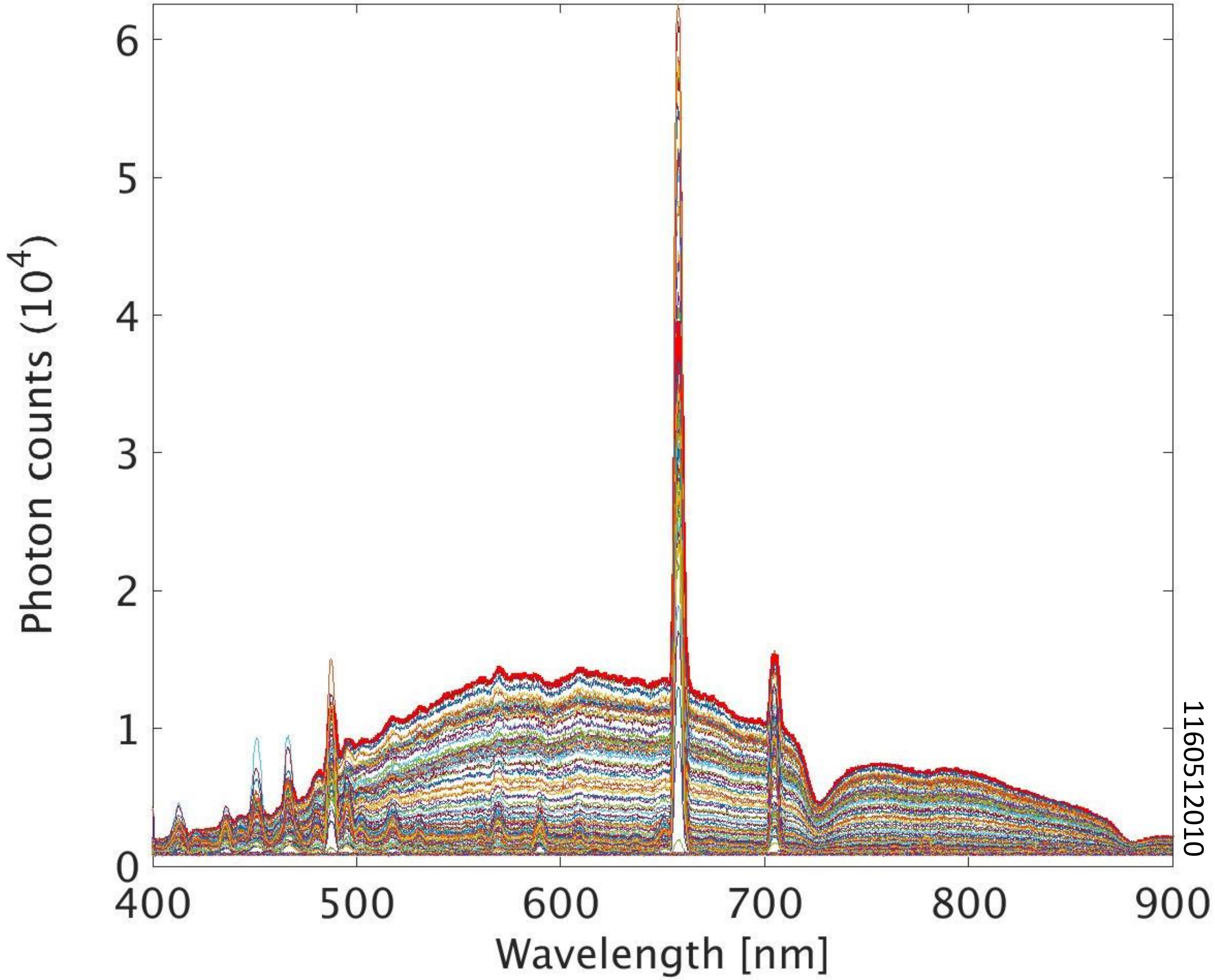

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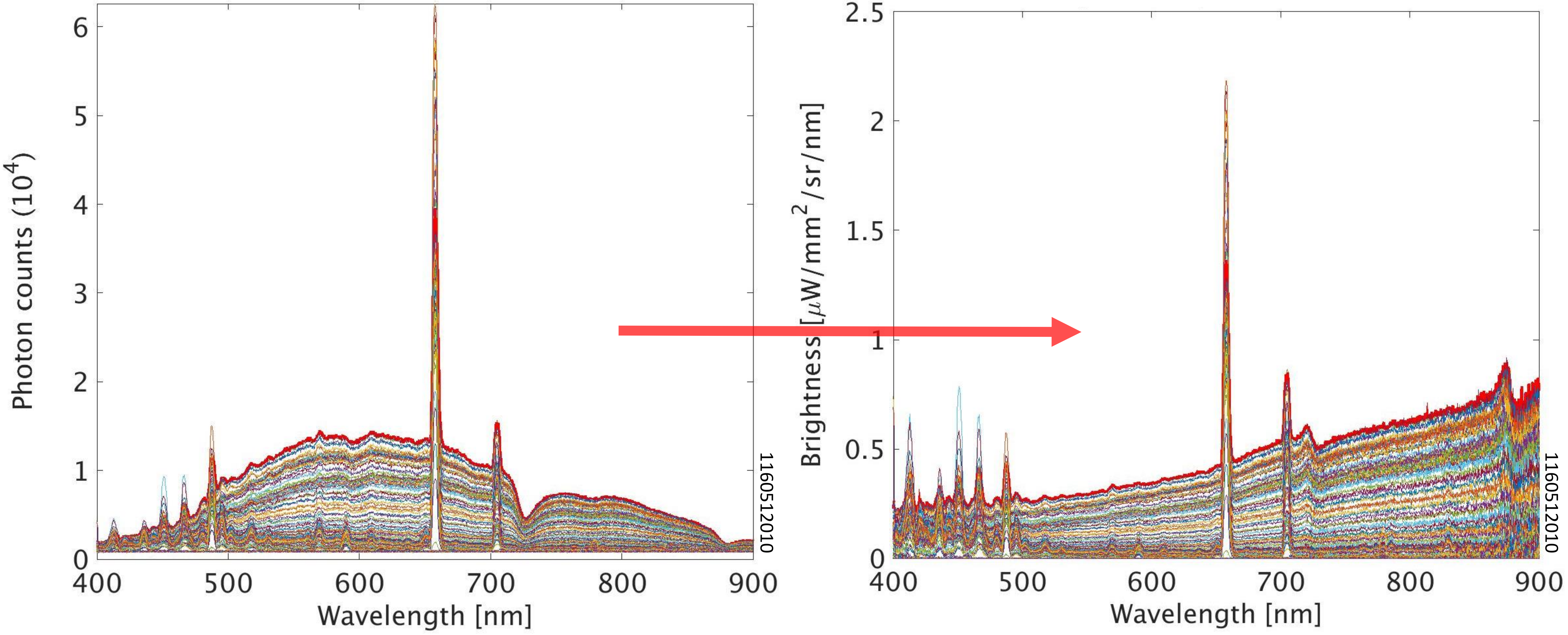
- $\bar{n}_e = 6.6 \cdot 10^{19}$ m^{-3}
- T_{e0} = 2.3 – 3.2 keV
- a_{beam} ≈ 7 cm (as seen by camera)
- V_{loop} ≈ 1.1 V

→ E = 0.26 V/m

→ **E/E_c ≈ 4.8**

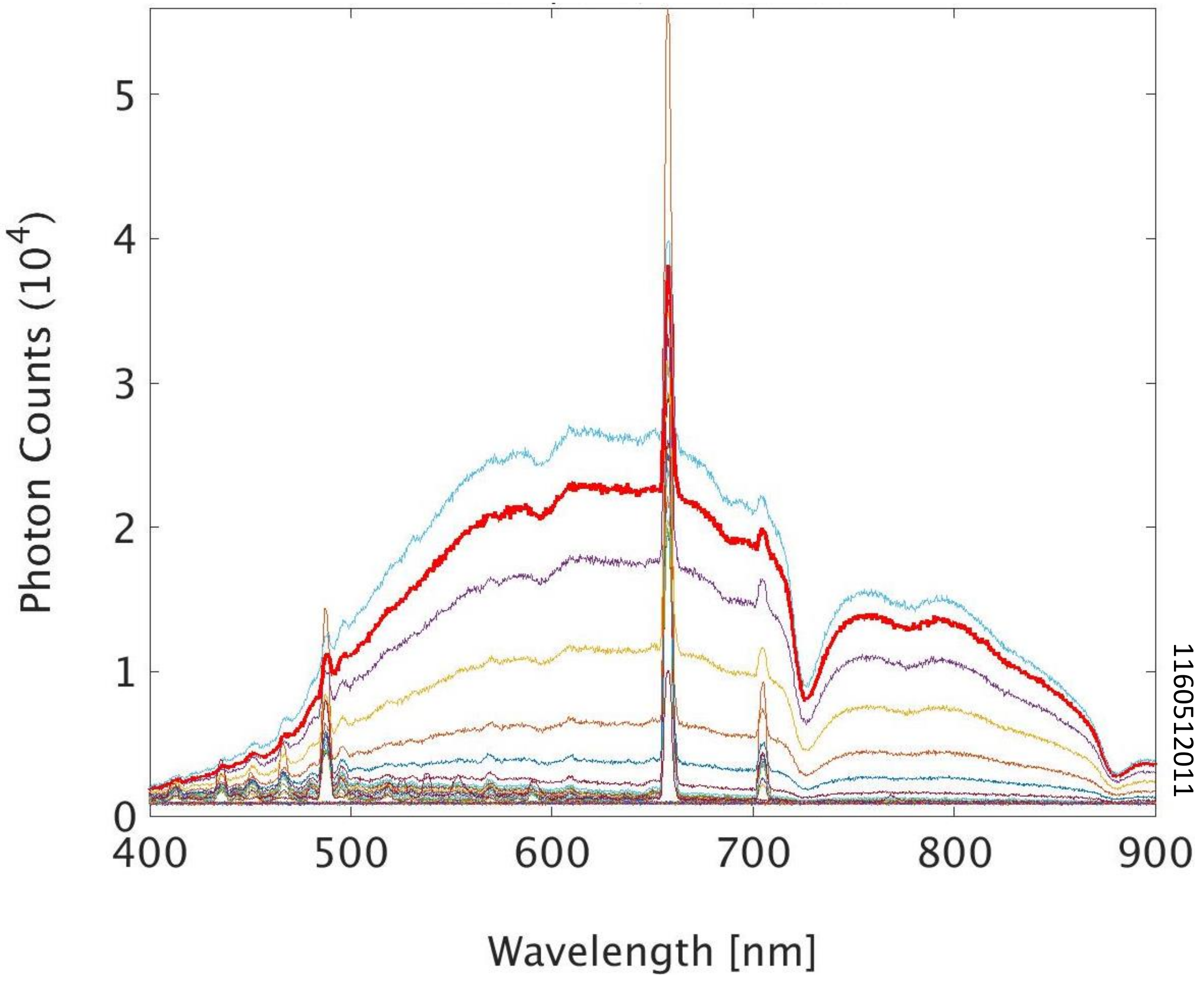
The red highlighted data is at t = 0.54 s and is used in this analysis.

Ramp-up synchrotron emission data



The red highlighted data is at t = 0.54 s and is used in this analysis.

Early synchrotron emission data



Plasma parameters at **t = 0.18 s**:

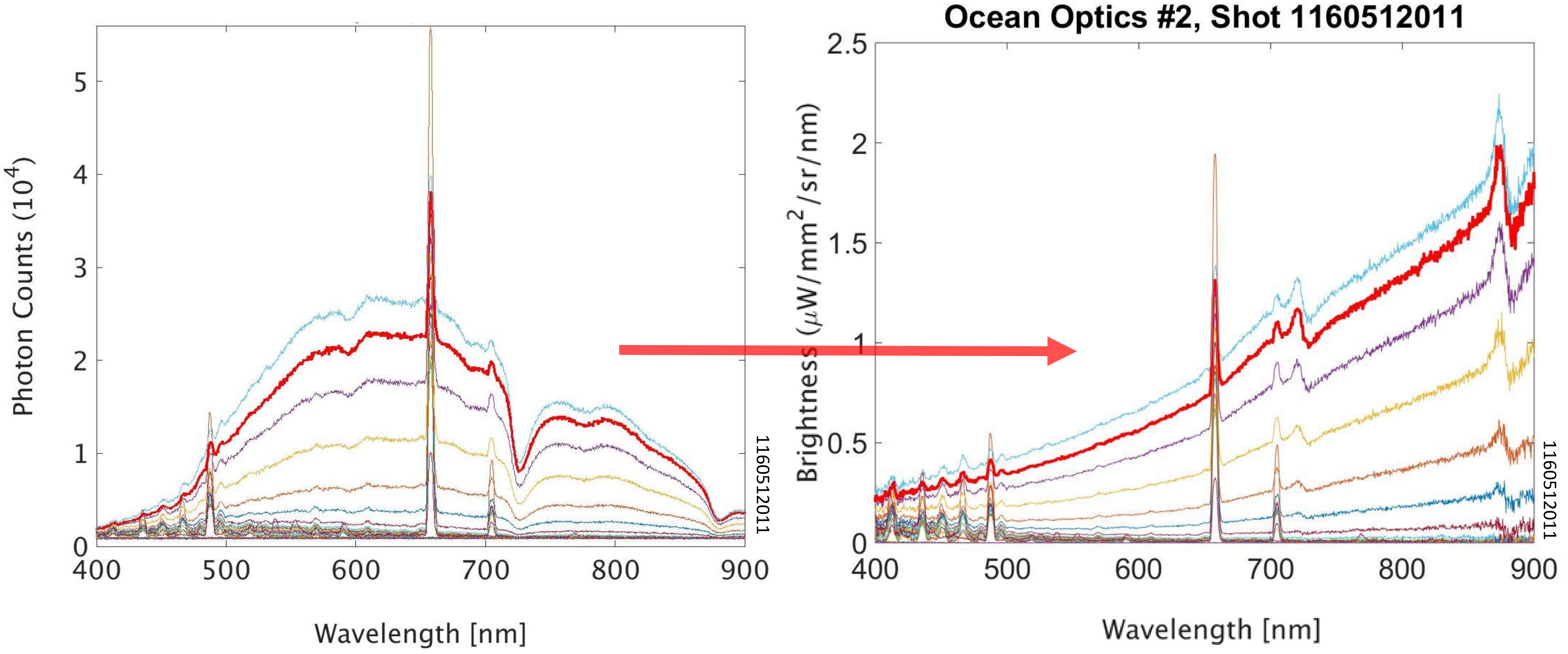
- B_t = 5.24 T
- I_p ≈ 670 kA
- $\bar{n}_e = 5.9 \cdot 10^{19}$ m^{-3}
- T_{e0} = 2.5 keV
- a_{beam} ≈ 6 cm (as seen by camera)
- V_{loop} ≈ 2.3 V

→ E = 0.54 V/m

→ **E/E_c ≈ 11**

The red highlighted data is at t = 0.18 s and is used in this analysis.

Early synchrotron emission data

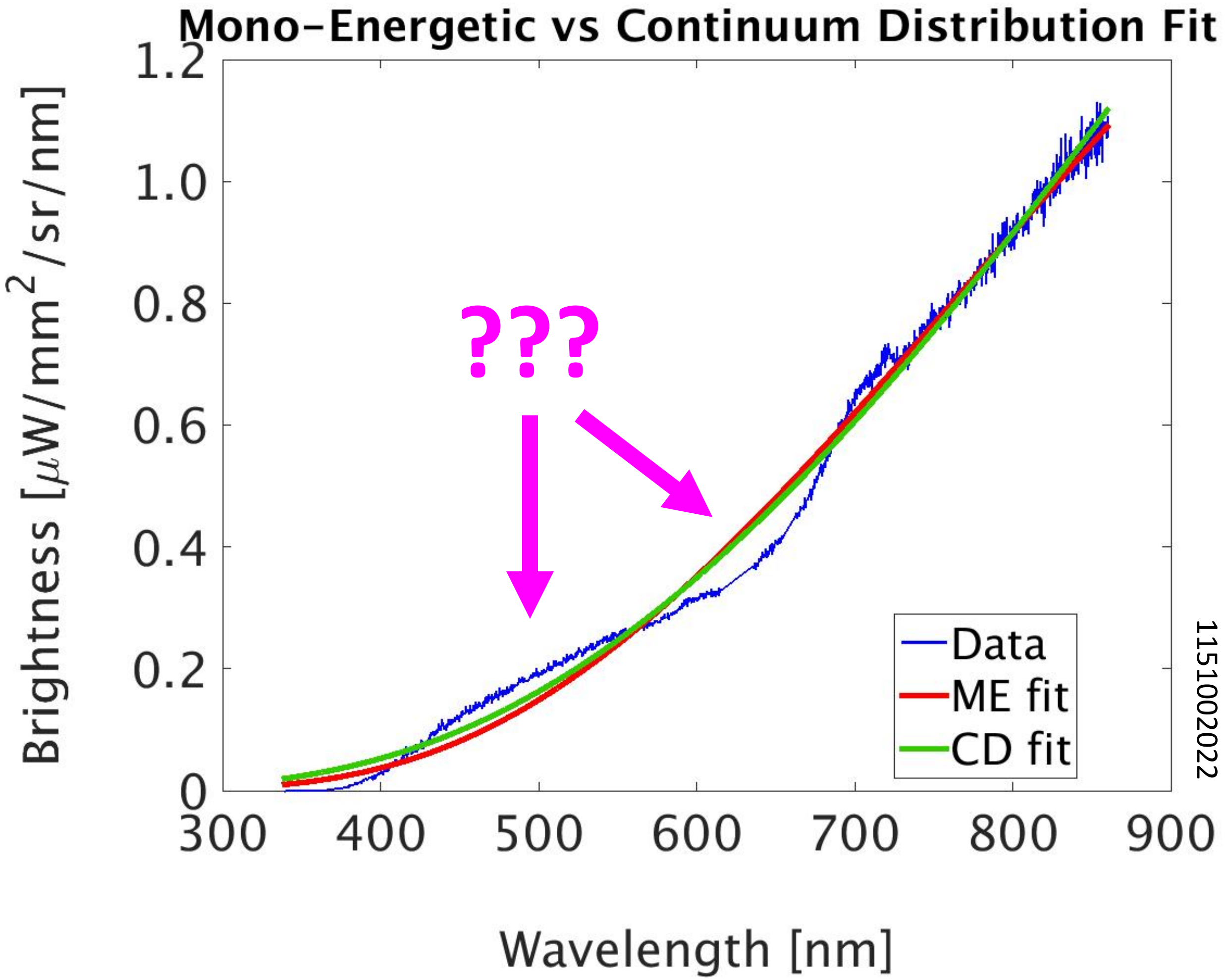


The red highlighted data is at t = 0.18 s and is used in this analysis.

Both flattop fits are comparable

- The **mono-energetic** and **continuum distribution** fits are very similar, with about the same goodness of fit.

- There is a **brightness feature** that cannot be fit by either.
  - Maybe we need a different RE distribution?
  - Or perhaps this is a result of a calibration error?

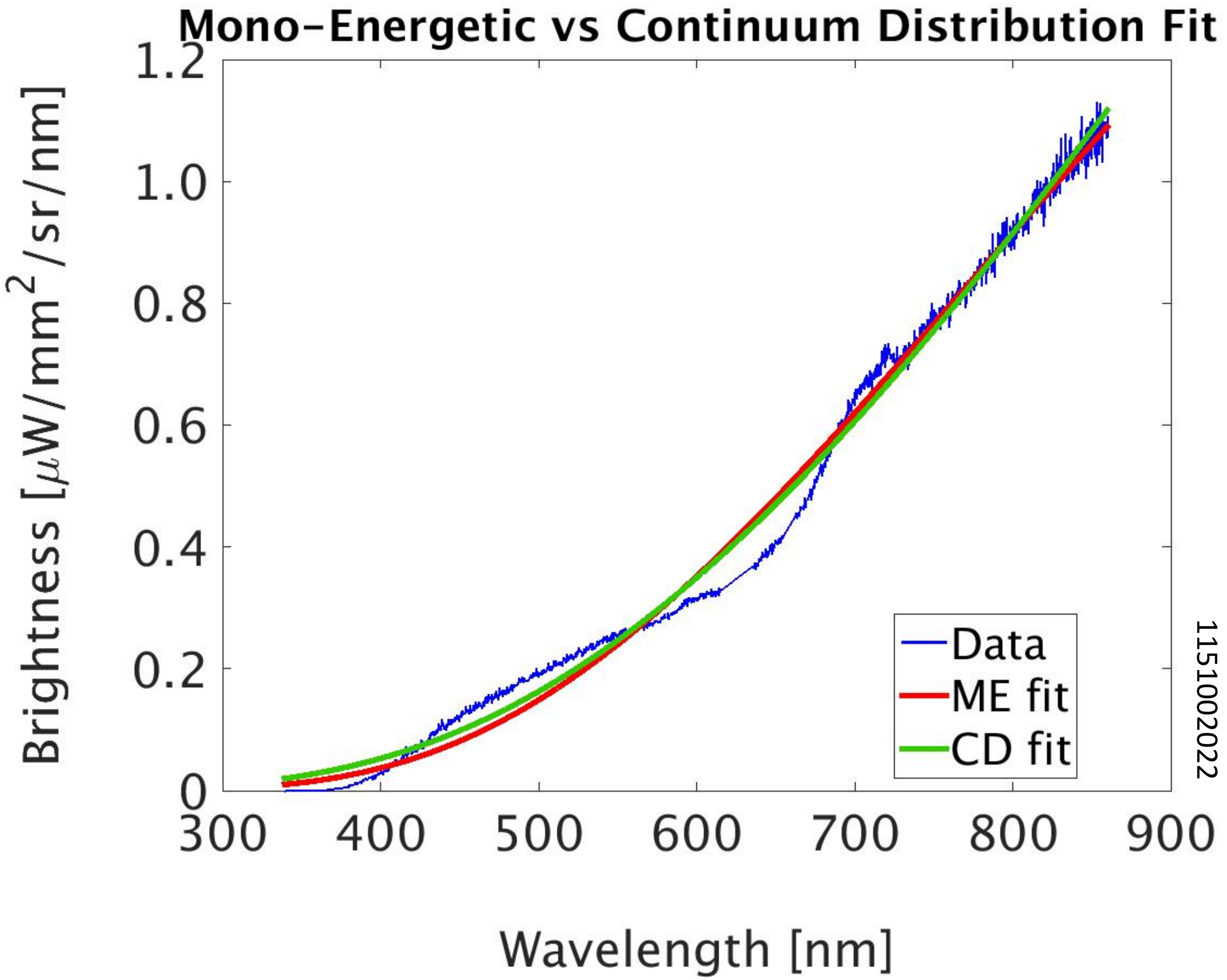


Both flattop fits are comparable

- resnorm = $\sum_{\lambda} [data(\lambda) - fit(\lambda)]^2$
  - Goodness of fit
  - MATLAB's *lsqcurvefit* was used to perform a nonlinear least squares fit to the two models.
  - We assume that each data point has the same uncertainty.

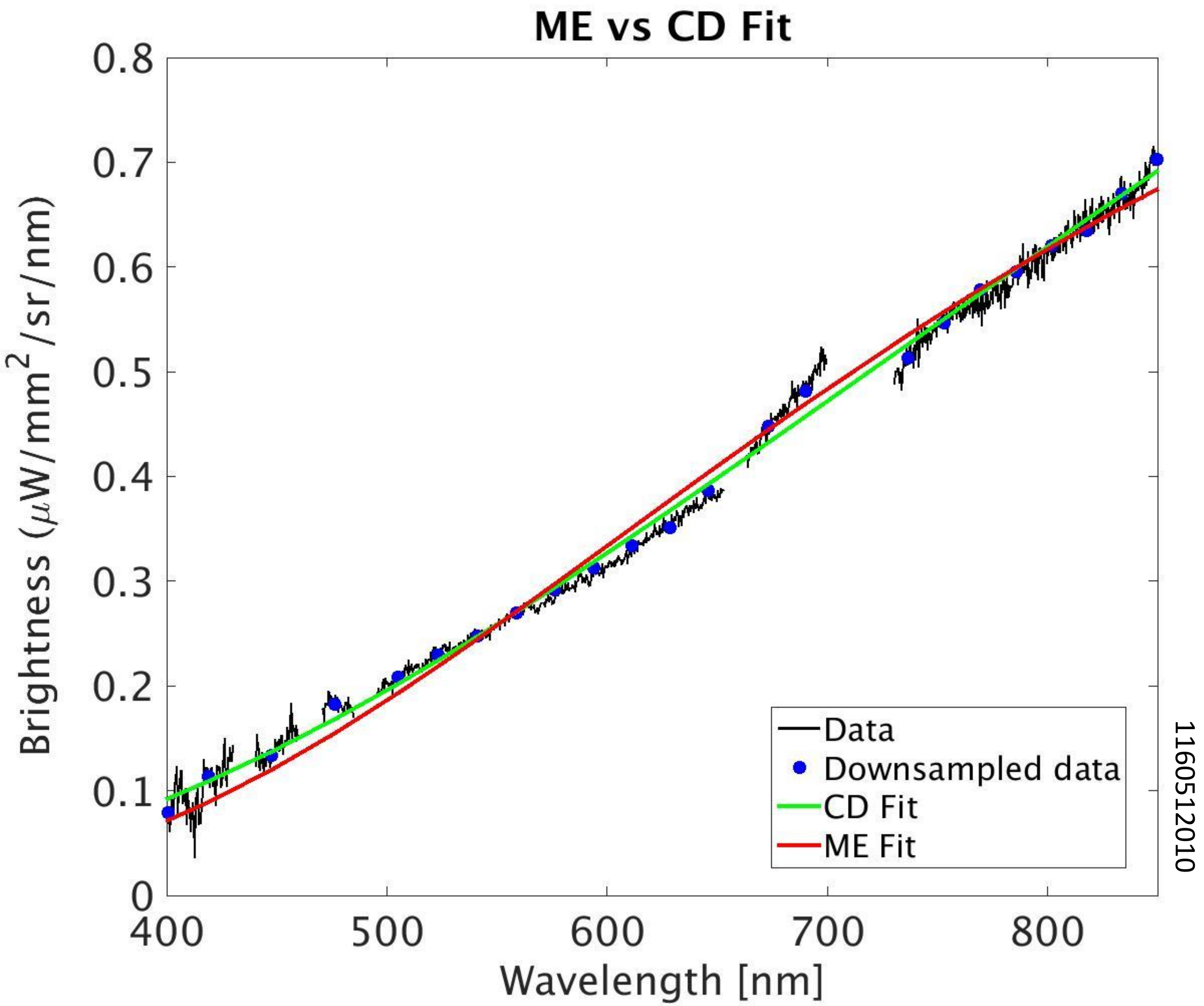
- **resnorm$_{mono}$ = 1.4 · 10^{-12}**

- **resnorm$_{dist}$ = 1.2 · 10^{-12}**



Both ramp-up fits are comparable

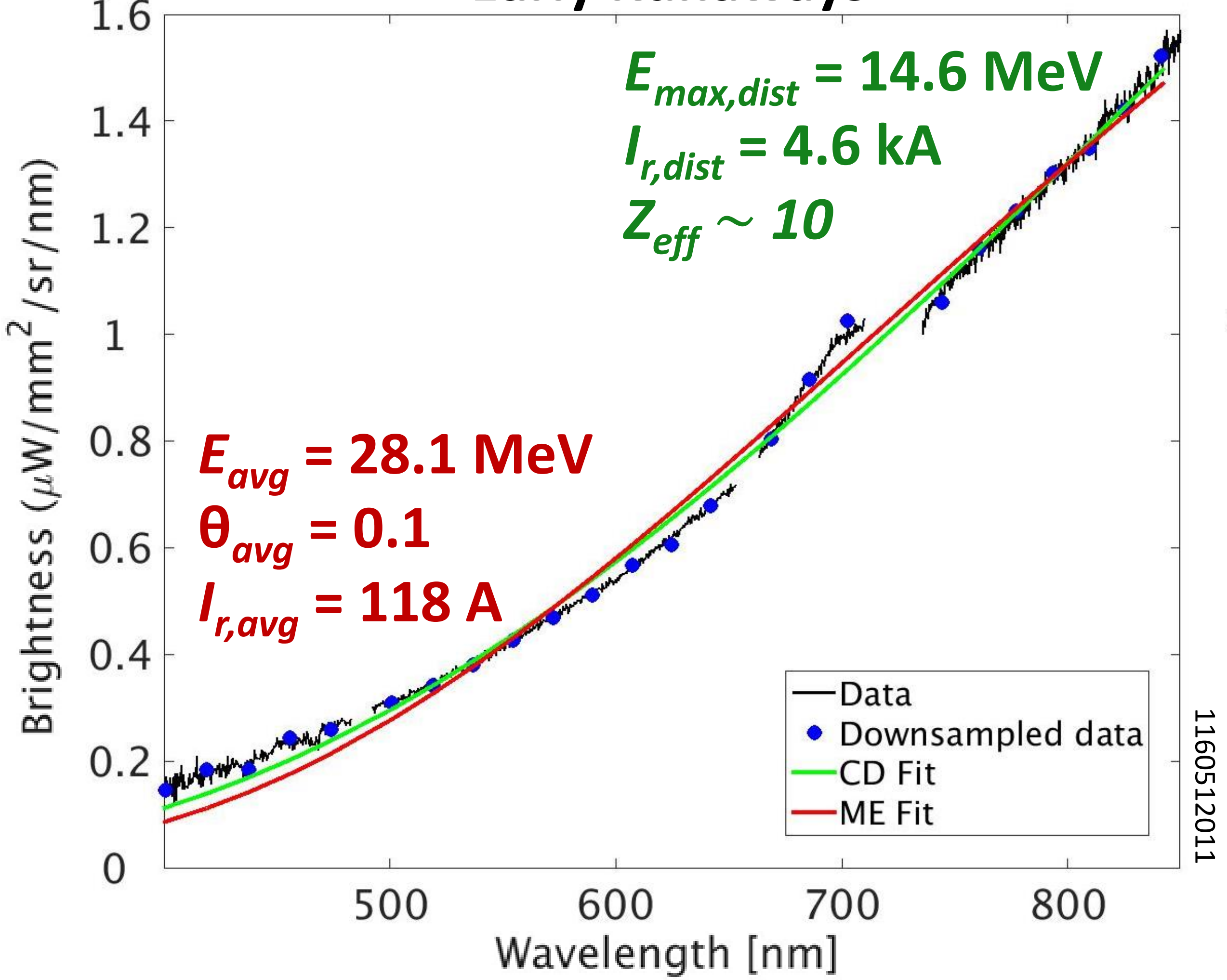
- The **mono-energetic** and **continuum distribution** fits are again very similar.
- E_{avg} = 30.2 MeV
- θ_{avg} = 0.1
- $I_{r,avg}$ = 40 A
- $E_{max,dist}$ = 12.6 MeV
- $I_{r,dist}$ = 933 A
- $T_{e,dist}$ = 3.2 keV



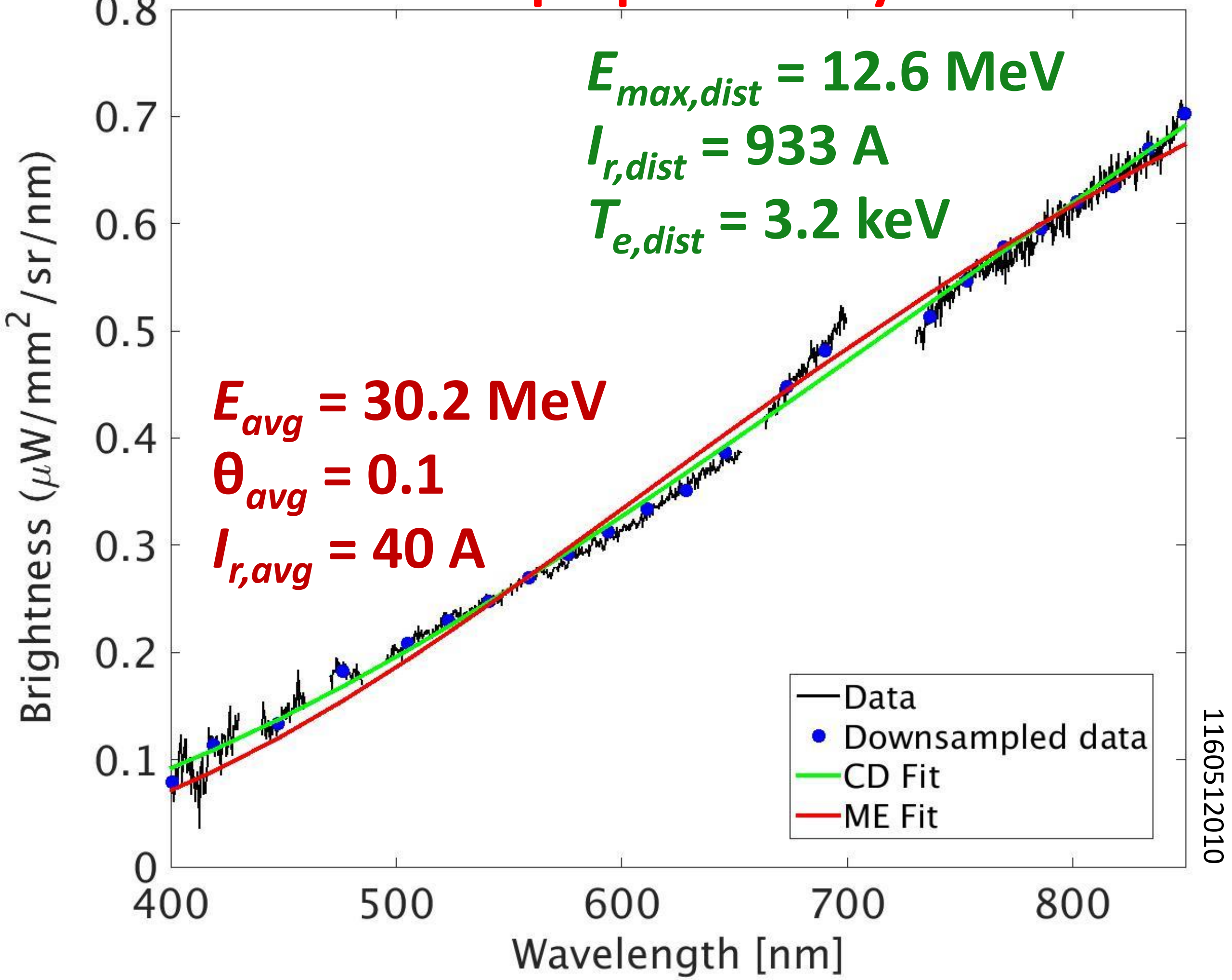
Alcator C-Mod

Both ME and CD fits are again comparable

Early Runaways

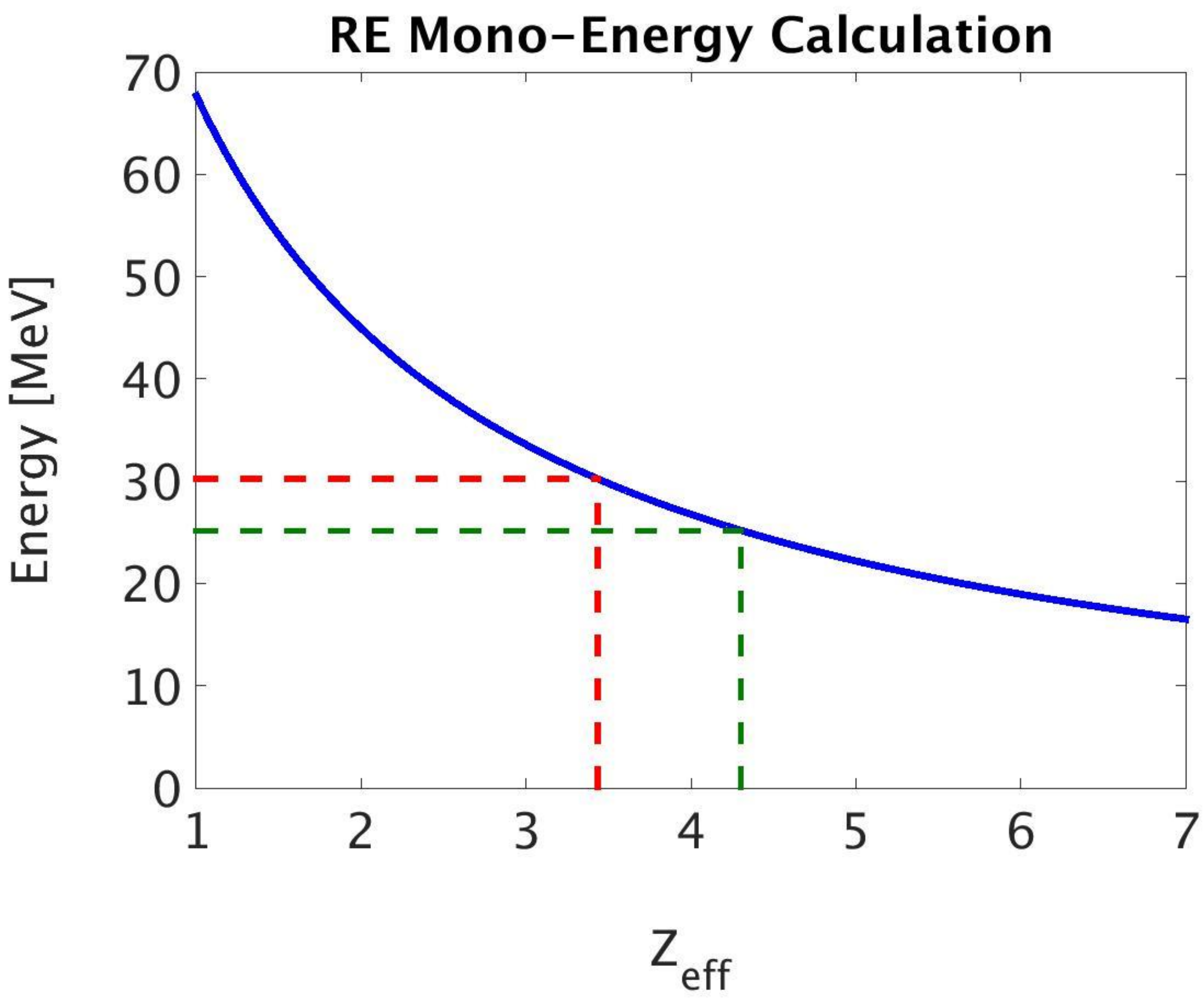


Ramp-up Runaways



"Bump on tail" formation

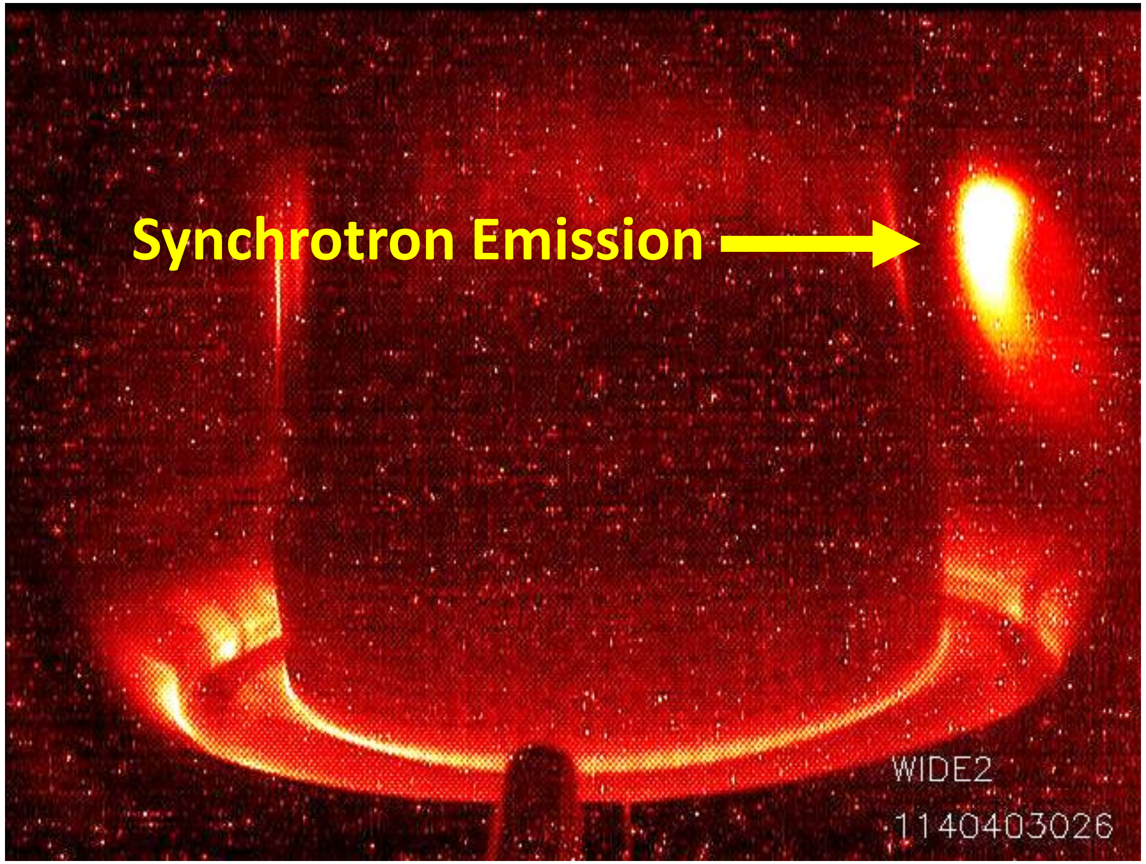
- In [1], the energy at which REs converge is calculated as a function of Z_{eff}, which we were not able to measure for the **flattop** data (shot 1151002022).
- For the plasma parameters at t = 1.5 s, a mono-energetic RE beam of 28 MeV is produced by a $Z_{eff,mono}$ of ~4 , which is consistent with experiments on C-Mod. [8]
- This also means that C-Mod's high Z_{eff} (3-7) in RE-producing plasma conditions could limit the RE energy to < 30 MeV.



[1] P. Aleynikov, et al. Phys. Rev. Lett. 114, 155001 (2015).
[8] R. S. Granetz, et al. Phys. Plasmas 21, 072506 (2014).

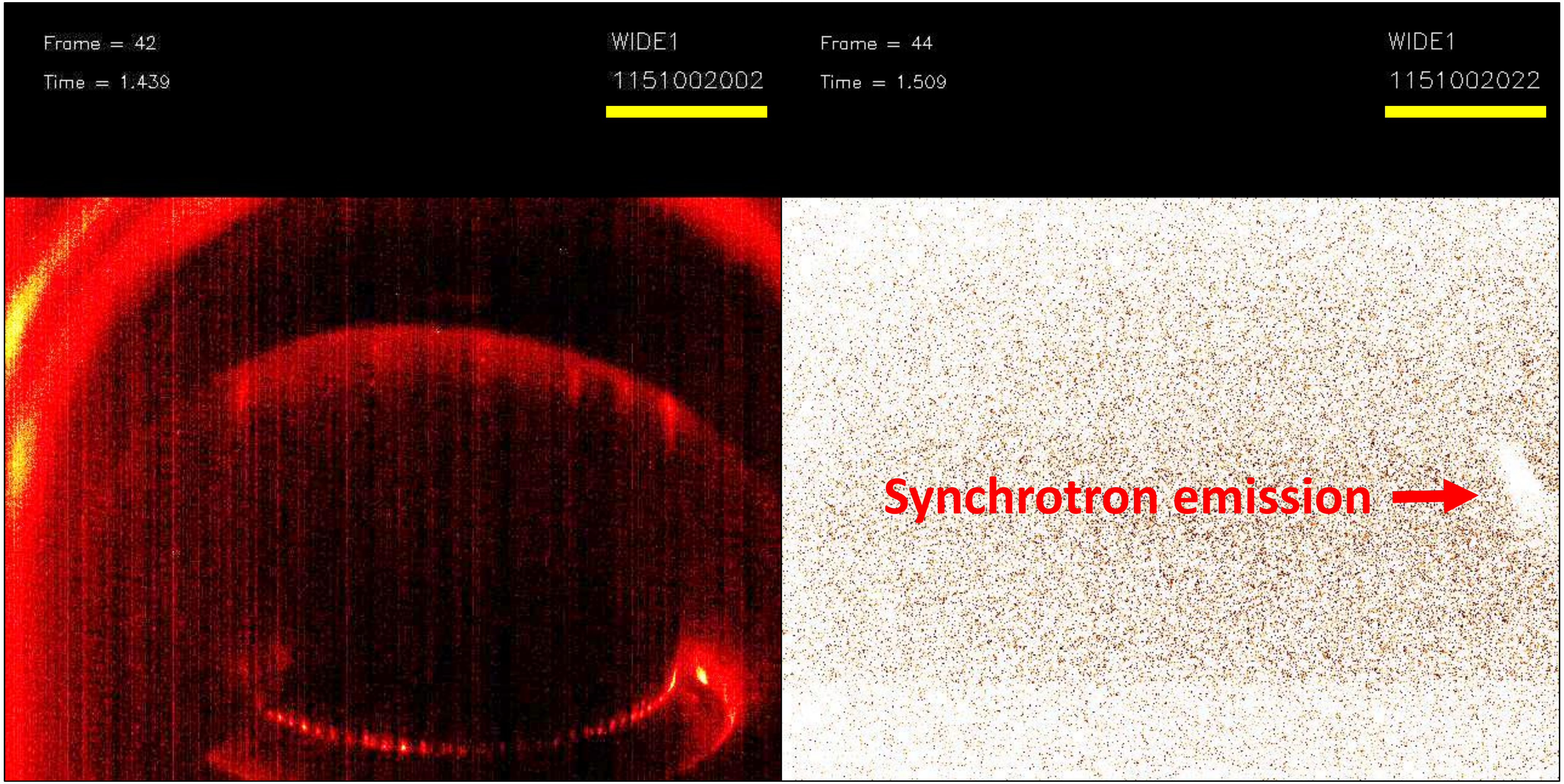
Runaway electrons

- In plasmas, the Coulomb collision frequency between particles varies as (*density*)/(*velocity*)^3.
- This can lead to a cascade of relativistic "runaway" electrons (REs) with energies of tens of MeV.
- Relativistic charged particles emit a cone of synchrotron radiation (SR) in their direction of motion.
- In C-Mod, this radiation can be in the visible/near-infrared range (300-900 nm).



Camera view inside Alcator C-Mod.

Camera view of SR



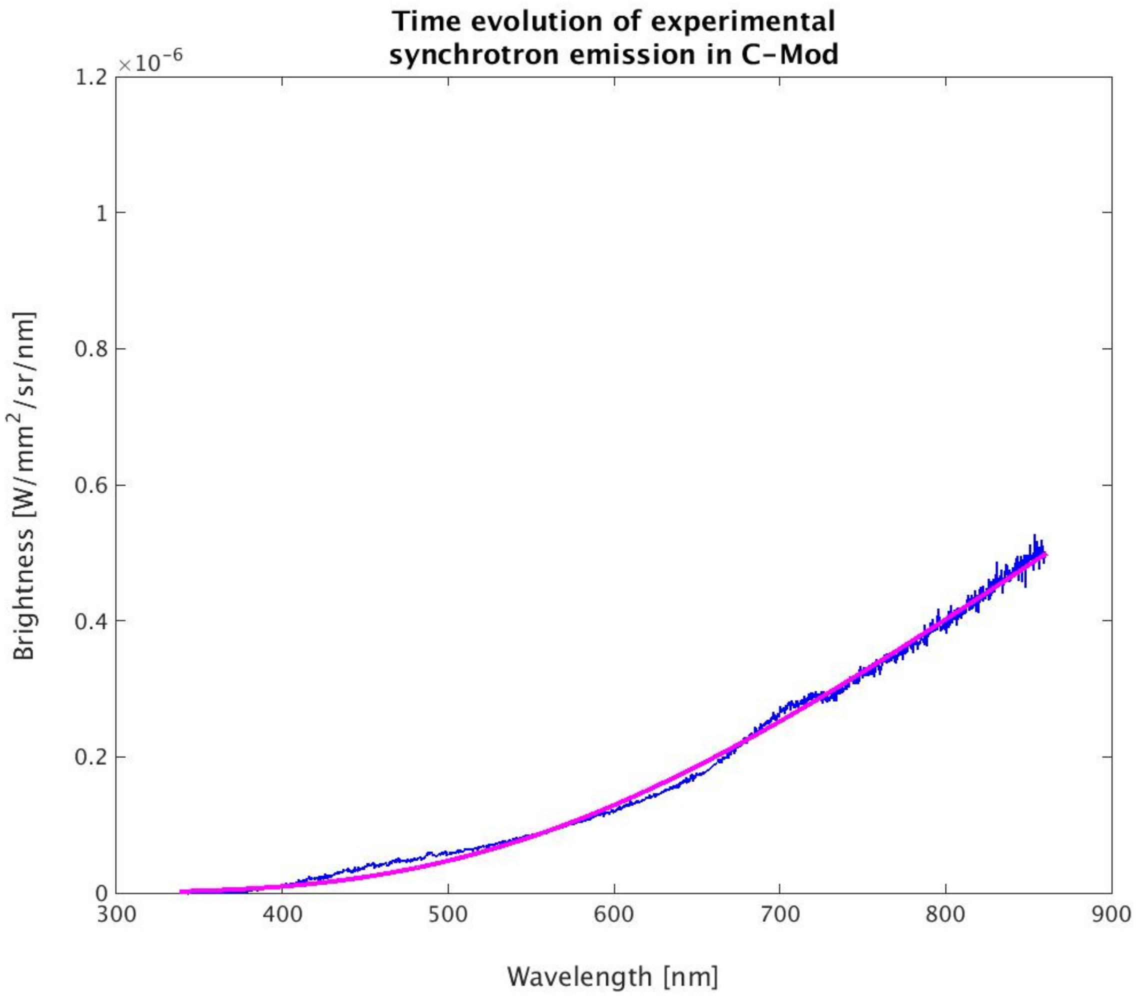
Camera view of SR

Parameters:

- R = 68 cm – C-Mod major radius
- a = 22 cm – C-Mod minor radius
- $a_{beam} \approx$ 5 cm – radius of RE beam
- $A_{beam} \approx$ 80 cm^2 – area of RE beam
- r_{lens} = 9 mm – lens aperture
- r_0 = 1.77 m – distance from lens to tangency radius (SR)

Time evolution: Mono-energetic RE beam

- t = 1.33s
  - E_{avg} = 24.3 MeV, σ_E = 1.8 MeV
  - θ_{avg} = 0.08, σ_ϑ = 0.02
  - I_{avg} = 48 A, σ_f = 2 A



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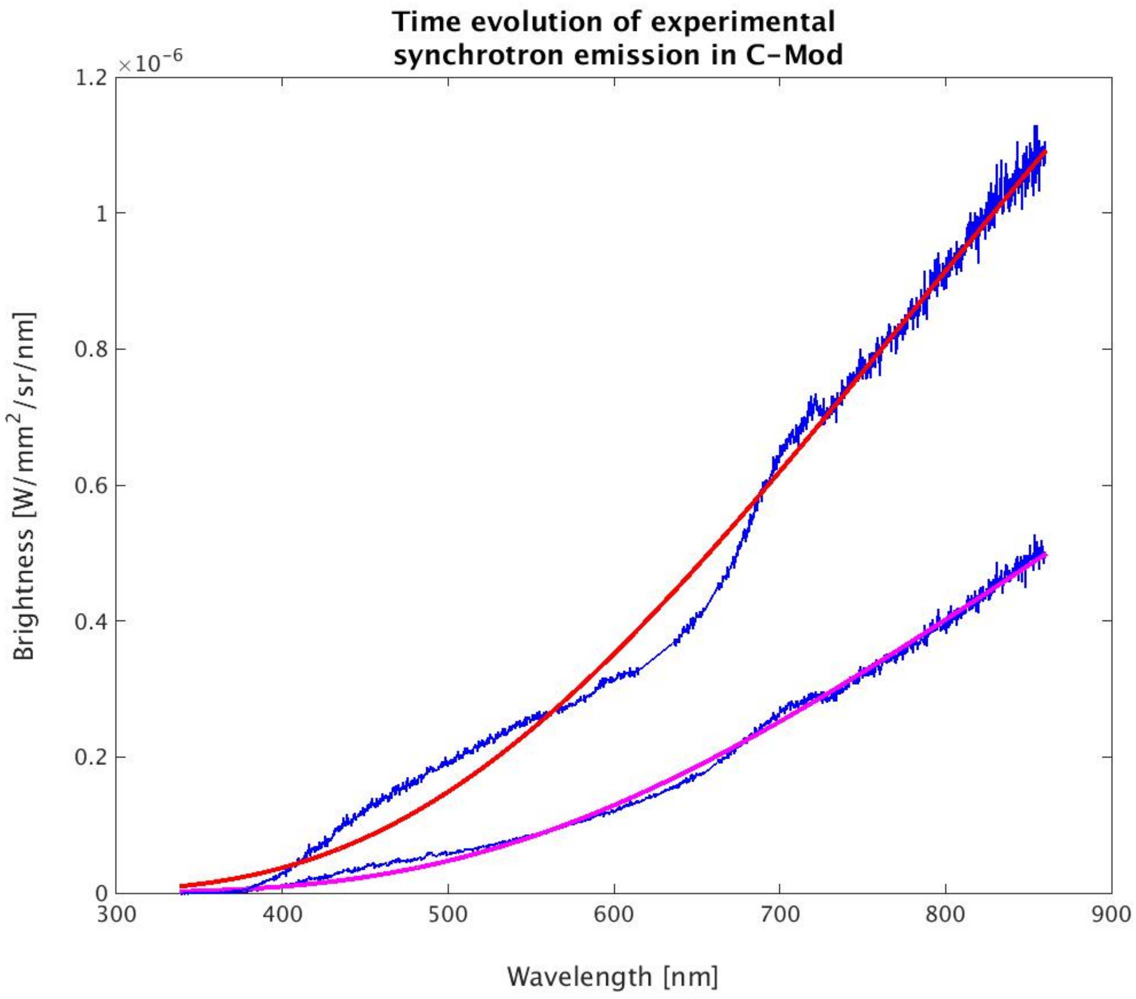
Time evolution: Mono-energetic RE beam

Alcator C-Mod

- t = 1.33s
  - $\boldsymbol{E_{avg}}$ **= 24.3 MeV,** $\boldsymbol{\sigma_E}$ **= 1.8 MeV**
  - θ_{avg} = 0.08, σ_ϑ = 0.02
  - $\boldsymbol{I_{avg}}$ **= 48 A,** $\boldsymbol{\sigma_f}$ **= 2 A**

- t = 1.50s
  - $\boldsymbol{E_{avg}}$ **= 28.0 MeV,** $\boldsymbol{\sigma_E}$ **= 1.2 MeV**
  - θ_{avg} = 0.09, σ_ϑ = 0.01
  - $\boldsymbol{f_{avg}}$ **= 81 A,** $\boldsymbol{\sigma_f}$ **= 1 A**



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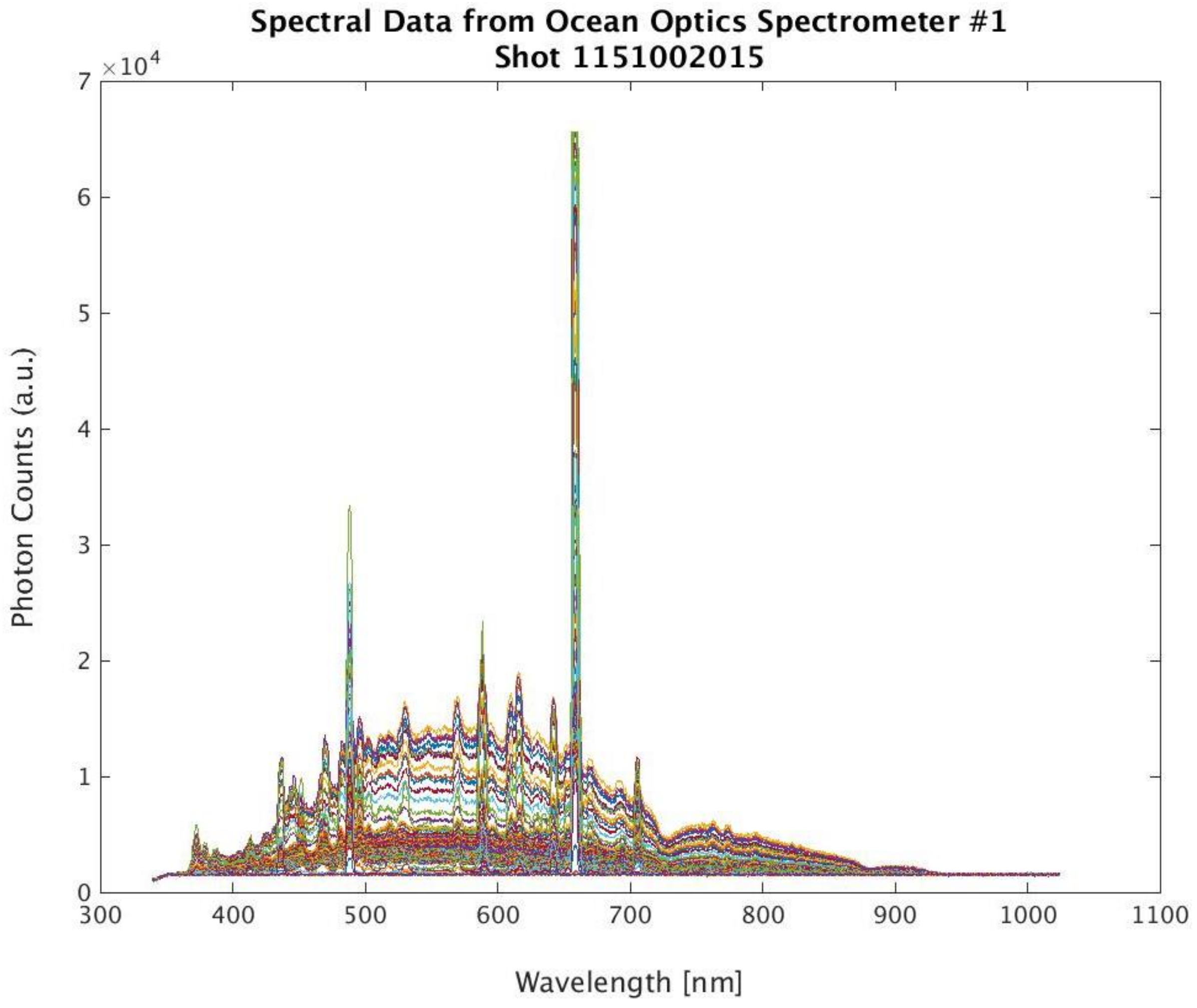
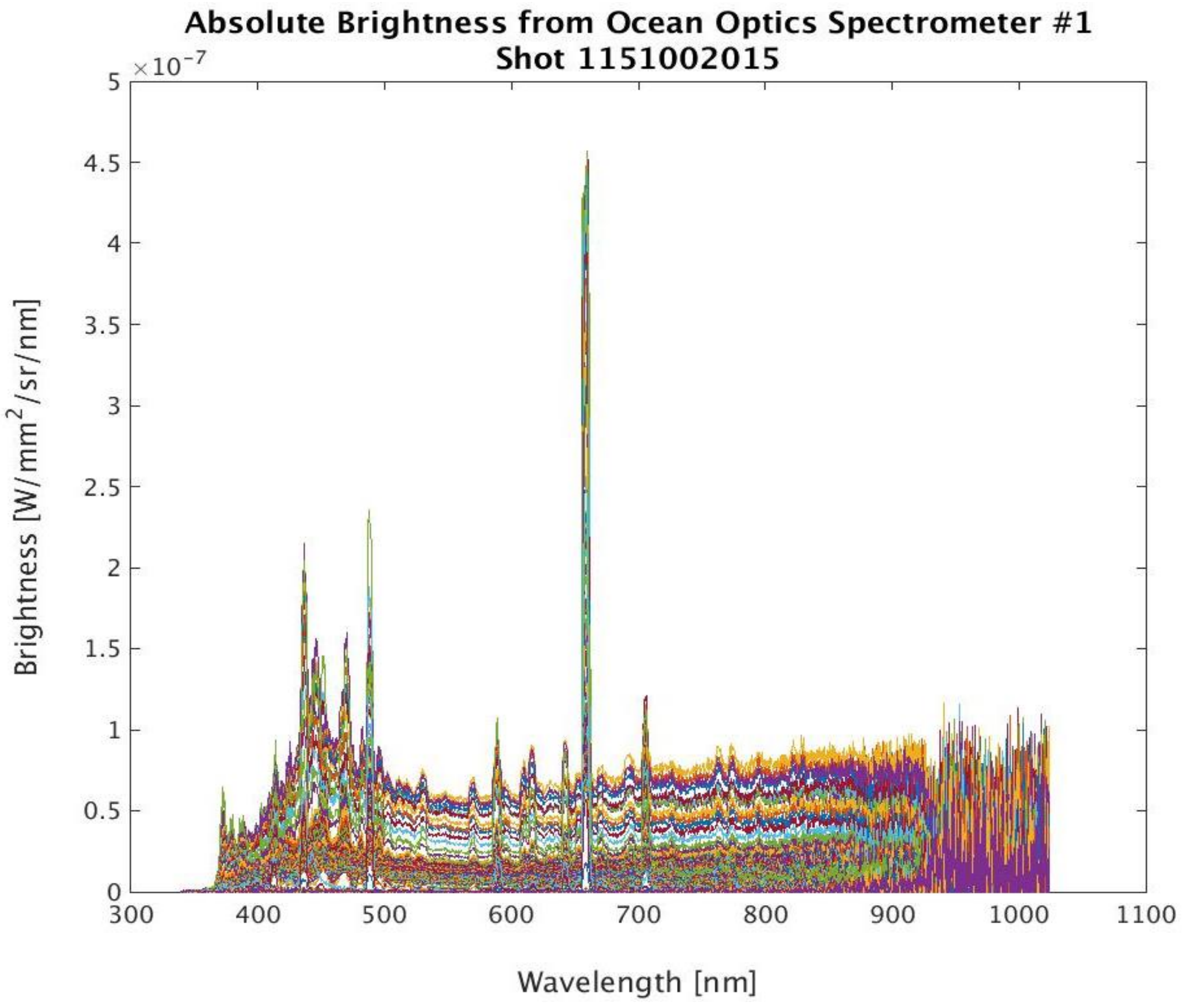
Non-monotonic brightness?





Full power calculation

Alcator C-Mod

The power radiated by a relativistic electron in a tokamak is given by [A]:

$$P_{full}(\lambda) = \frac{ce^2}{\epsilon_0 \lambda^3 \gamma^2} \{ \int_0^\infty \frac{1+2y^2}{y} J_0(ay^3) \sin\left(\frac{3}{2}\xi\left(y+\frac{1}{3}y^3\right)\right) dy$$

$$+ \frac{4\eta}{1+\eta^2} \int_0^\infty y\, J_1(ay^3) \cos\left(\frac{3}{2}\xi\left(y+\frac{1}{3}y^3\right)\right) dy \; - \frac{\pi}{2} \}$$

where

$$a = \xi\eta/(1+\eta^2), \quad \xi = \frac{4\pi}{3} \frac{R}{\lambda\gamma^3\sqrt{1+\eta^2}}, \quad \eta \cong \frac{eB}{m} \frac{R}{\gamma c} \frac{v_\perp}{v_\parallel}$$

and $\frac{v_\perp}{v_\parallel}$ is the pitch and γ = E/mc^2 is the relativistic Lorentz factor.

[A] Equation 2 in A. Stahl, et al. "Synchrotron radiation from a runaway electron distribution in tokamaks." 2013.

Power density approximation

- Using the approximation:

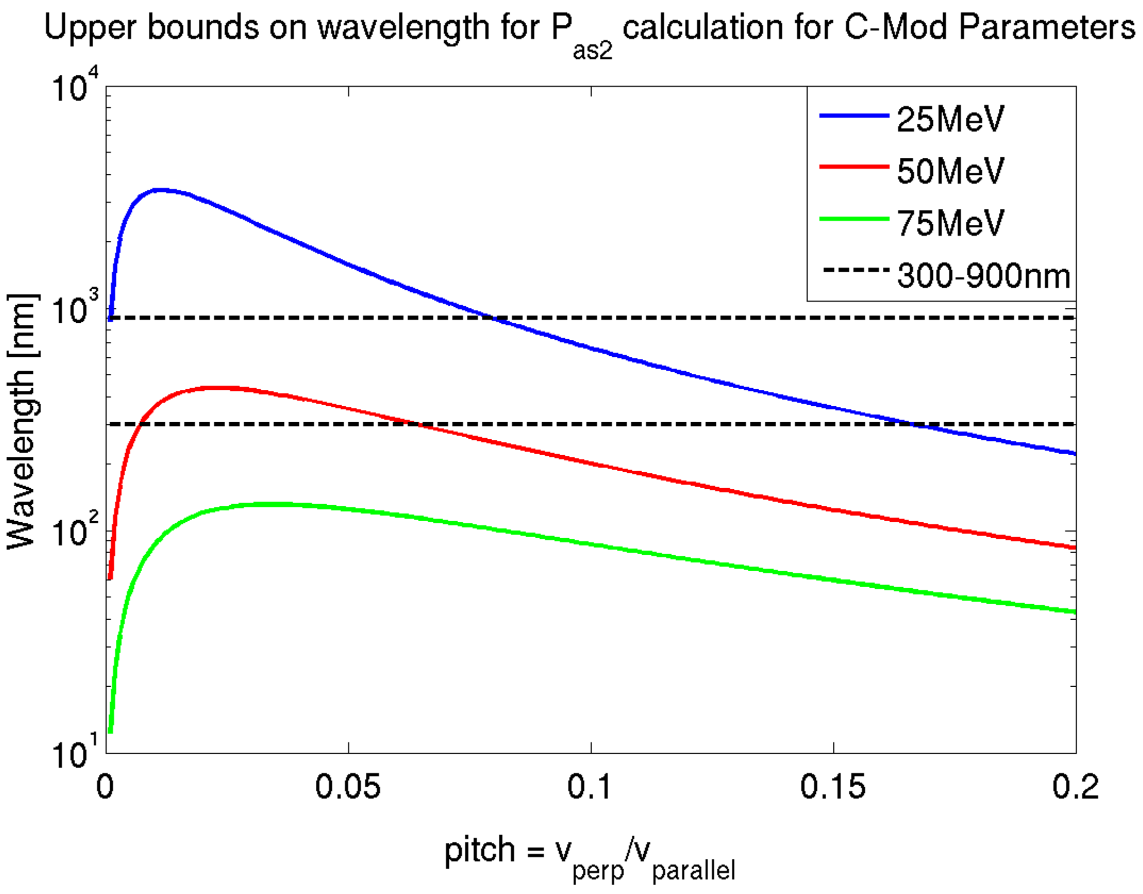
$$\lambda \ll \frac{4\pi}{3} R\eta/[\gamma^3(1+\eta)^3]$$

the power calculation reduces to [B] :

$$P_{as2}\,(\lambda) = \frac{\sqrt{3}}{8\pi}\frac{ce^2\gamma}{\epsilon_0\lambda^2 R}\frac{(1+\eta)^2}{\sqrt{\eta}}\exp\left(-\frac{4\pi}{3}\frac{R}{\lambda\gamma^3}\frac{1}{1+\eta}\right)$$

- This approximation is only valid for C-Mod at low energies (~25MeV).

[B] Equation 7 in Stahl, et al., (2013).



Mono-energetic brightness

For a mono-energetic (mono-pitch) beam, the brightness ($W/m^3/sr$) is [C]:

$$B(\lambda, \theta_{eff}, \gamma) = \frac{2\, R\, n_r}{\pi\, \theta_{eff}} P(\lambda, \theta_{eff}, \gamma)$$

where

$$\theta_{eff} \approx \sqrt{\left(\frac{v_\perp}{v_\parallel}\right)^2 + \gamma^{-2} + \left(\frac{r_{lens}}{r_0}\right)^2}$$

is the effective viewing aperture and n_r is the runaway beam density at this energy.

[C] Equation 10 in Stahl, et al., 2013.

Distribution of energies and pitches

For a distribution of energies and pitch angles [D]:

$$f_{RE}(p_{\parallel}, p_{\perp}) = \frac{n_r \hat{E}}{2\pi c_z p_{\parallel} ln\Lambda} \exp\left(-\frac{p_{\parallel}}{c_z ln\Lambda} - \frac{\hat{E} p_{\perp}^2}{2 p_{\parallel}}\right)$$

The brightness is calculated [E]:

$$B(\lambda) = 4R \int_0^1 \int_{p\_min}^{p\_max} \frac{1}{\theta_{eff}(\chi)} P\left(\lambda, \theta_{eff}(\chi), \gamma(p)\right) f_{RE}(p, \chi) p^2 dp d\chi$$

[D,E] Equations 9 and 12, Stahl, et al., 2013.

Acknowledgments

Many thanks to Adam Stahl and the entire Chalmers Plasma Theory group for CODE, debugging, and fruitful discussions on runaway evolution.

Dr. Bob Mumgaard is gratefully acknowledged for his assistance in absolutely calibrating the Ocean Optics spectrometers and calculating the brightness.